Work, energy and power

Work done by a force, conservation of energy, efficiency, power and P = Fv, and deriving and using the formulas for gravitational potential energy and kinetic energy.

Energy conservation

Work done

$$W = Fs$$

Work done is force × displacement **in the direction of the force**. If the force is at angle θ to the displacement, $W = Fs\cos\theta$. The unit is the joule. A force at right angles to the motion does no work, for example the centripetal force on an object in a circle.

Conservation of energy

> **Energy cannot be created or destroyed. It can only be transferred from one store to another, so the total energy of a closed system stays constant.**

Work done is how energy is transferred by a force.

Efficiency

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} = \frac{\text{useful power output}}{\text{total power input}}$$

Efficiency is often given as a percentage, and it can never exceed 100%. The rest of the energy usually ends up as thermal energy in the surroundings.

TRY IT

A motor lifts 40 kg through 3.0 m in 5.0 s while taking 400 W of electrical power. Find its efficiency. Useful power $= mgh/t = 40 \times 9.81 \times 3.0/5.0 = 235$ W. Efficiency $= 235/400 = 0.59$, or 59%.

Power

Power is the rate of doing work, or the work done per unit time:

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$$P = \frac{W}{t}$$

The unit is the watt (1 W = 1 J/s).

Deriving P = Fv

A constant force F moves an object through a displacement s in time t. Then $P = W/t = Fs/t$, and $s/t = v$, so:

$$P = Fv$$

> TRY IT
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> **A car travels at a steady 30 m/s against a total resistive force of 800 N. Find the power of its engine.** At steady speed, the driving force equals the resistance. $P = 800 \times 30 = 24$ kW.

If the car is accelerating, the driving force is greater than the resistance. Use $F_{\text{driving}} = P/v$, then $F_{\text{driving}} - F_{\text{resistance}} = ma$.

Gravitational potential energy and kinetic energy

Gravitational potential energy

To lift a mass m at steady speed through a height Δh, the force needed equals the weight, mg. The work done is force × distance:

$$\Delta E_P = mg\Delta h$$

This holds only where the field is uniform, which means small height changes near the Earth's surface. Chapter 13 deals with larger distances.

Kinetic energy

Take a mass m accelerated from rest by a force F through a distance s, reaching speed v. The work done is $W = Fs = (ma)s$. From $v^2 = u^2 + 2as$ with $u = 0$, $as = \frac{1}{2}v^2$, so:

$$E_K = \frac{1}{2}mv^2$$

Kinetic energy depends on the square of the speed, so doubling the speed makes it four times as large.

Using energy to solve problems

When there is no friction or drag, the loss of gravitational potential energy equals the gain of kinetic energy, so $mg\Delta h = \frac{1}{2}mv^2$ and $v = \sqrt{2g\Delta h}$. The mass cancels.

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A 60 kg skier starts from rest and descends a vertical height of 20 m, reaching 15 m/s at the bottom. Find the work done against friction. GPE lost $= 60 \times 9.81 \times 20 = 11\,770$ J. KE gained $= \frac{1}{2}(60)(15^2) = 6750$ J. Work against friction $= 11\,770 - 6750 = 5.0$ kJ.

NOTE

Exam tip: energy methods do not need the shape of the path, so they often give the answer in one line where forces and the equations of motion would take several. When friction acts, always write: energy lost = energy gained + work done against friction.

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