Superposition

The principle of superposition, stationary waves on strings, in air columns and with microwaves, nodes and antinodes, diffraction, coherence and two-source interference, and the diffraction grating.

Stationary waves

The principle of superposition

When two or more waves meet at a point, the resultant displacement is the vector sum of their individual displacements at that point.

How a stationary wave forms

A stationary wave forms when two progressive waves of the same frequency and similar amplitude, travelling in **opposite directions**, overlap. Usually a wave reflects from a boundary and meets itself.

- **Nodes** are points of zero amplitude, where the two waves always cancel.
- **Antinodes** are points of maximum amplitude, where the two waves always reinforce.

The distance between neighbouring nodes, or between neighbouring antinodes, is $\lambda/2$. A node and the next antinode are $\lambda/4$ apart. Measure the node spacing to find the wavelength.

	Progressive wave	Stationary wave
Energy	transferred along the wave	not transferred
Amplitude	same everywhere	varies from zero to maximum
Phase	changes continuously with position	all points between neighbouring nodes are in phase
Wave profile	moves along	stays in place

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Demonstrating stationary waves

- **Stretched string:** a vibration generator drives one end of a string with the other end fixed, and at particular frequencies a stationary wave appears, with nodes at both fixed ends. The lowest frequency has one loop, so $L = \lambda/2$.
- **Microwaves:** a transmitter faces a metal plate, which reflects the waves back. A detector moved between them finds alternate maxima and minima, $\lambda/2$ apart.
- **Air column:** a loudspeaker sounds above a tube closed at one end. At certain frequencies the air column resonates. There is a **node at the closed end** and an **antinode at the open end**, so the first resonance is at $L = \lambda/4$ and the next at $L = 3\lambda/4$. For a tube open at both ends, the first resonance is at $L = \lambda/2$.

> TRY IT
>
> **A string 0.90 m long vibrates with three loops at 120 Hz. Find the wave speed.** Three loops means $3\lambda/2 = 0.90$, so $\lambda = 0.60$ m. $v = f\lambda = 120 \times 0.60 = 72$ m/s.

Diffraction

Diffraction is the spreading of a wave as it passes through a gap or round an edge.

The effect depends on the gap width compared with the wavelength. Spreading is greatest when the gap is about equal to the wavelength, and a much wider gap gives little spreading. You can see this in a ripple tank with straight wavefronts and an adjustable gap. Note that the wavelength does not change.

Interference

Interference is the superposition of waves from two or more sources, which gives a pattern of reinforcement and cancellation.

Coherent sources have a **constant phase difference**, which means they must have the same frequency.

Conditions for visible fringes

For a two-source pattern to be observed:

1. The sources must be coherent.
2. The waves must have similar amplitudes, for good contrast.
3. For transverse waves, they must be polarised in the same plane or be unpolarised.
4. The waves must overlap.

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Path difference

For sources in phase:

Path difference	Phase difference	Result
$n\lambda$	0	constructive: maximum
$(n+\frac{1}{2})\lambda$	$180°$	destructive: minimum

Demonstrating interference

- **Water waves:** two dippers on the same bar in a ripple tank.
- **Sound:** two loudspeakers on the same signal generator. Walk along a line and hear it get louder and quieter.
- **Microwaves:** one transmitter behind a metal plate with two slits, and a detector moved across.
- **Light:** a laser through a double slit onto a screen. Two separate lamps are not coherent, so they cannot be used.

Young's double slit

$\lambda = \frac{ax}{D}$

where a is the slit separation, x is the fringe separation, and D is the distance from the slits to the screen.

> TRY IT
>
> **Slits 0.50 mm apart give fringes 2.4 mm apart on a screen 2.0 m away. Find the wavelength.**
> $\lambda = (0.50 \times 10^{-3})(2.4 \times 10^{-3})/2.0 = 6.0 \times 10^{-7}$ m.

Measure across several fringes and divide, which reduces the percentage uncertainty.

The diffraction grating

A **diffraction grating** has a very large number of parallel slits, close together. It gives bright, sharp maxima at certain angles:

$d \sin\theta = n\lambda$

where d is the slit spacing and n is the order of the maximum.

A grating labelled "N lines per metre" has $d = 1/N$. Since $\sin\theta$ cannot exceed 1, the highest order you can see is the largest whole number $n \le d/\lambda$.

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Finding the wavelength of light

Shine a laser at right angles onto a grating of known d. Measure the distance D to the screen and the distance y from the central maximum to the first-order maximum. Then $\tan\theta = y/D$, and $\lambda = d\sin\theta$. Measuring both sides and using higher orders gives better accuracy.

TRY IT

Light of 600 nm falls on a grating with 500 lines per mm. Find the angle of the first order and the highest order. $d = 1/(5.0\times10^{5}) = 2.0\times10^{-6}$ m. $\sin\theta = 6.0\times10^{-7}/2.0\times10^{-6} = 0.30$, so $\theta = 17°$. $d/\lambda = 3.33$, so the highest order is $n = 3$.

NOTE

Exam tip: gratings have larger angles and sharper maxima than a double slit, which makes the wavelength more accurate. Do not use the small-angle formula $\lambda = ax/D$ with a grating.

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