Stars and the Hertzsprung-Russell diagram

Black body radiation and Wien's law, luminosity and intensity, parallax, the Hertzsprung-Russell diagram and the life cycle of a star, and redshift.

Black body radiation

A **black body** absorbs all the radiation that falls on it and, when hot, emits a continuous spectrum whose shape depends only on its temperature. Stars are close to black bodies.

As the temperature rises, the total power emitted rises steeply and the **peak wavelength** gets shorter. **Wien's law** gives the peak:

$$\lambda_{\max} T = 2.898 \times 10^{-3}\ \text{m K}$$

> **TRY IT**
>
> The Sun's radiation peaks at 500 nm. Find its surface temperature.

$$T = \frac{2.898 \times 10^{-3}}{500 \times 10^{-9}} = 5800\ \text{K}.$$

> **NOTE**
>
> A hotter star looks bluer and a cooler star looks redder. Reading the peak of the curve is how temperatures are found.

Luminosity and intensity

The **luminosity** L of a star is the total power it radiates (W). The **intensity** I received at distance d is the power per unit area:

$$I = \frac{L}{4\pi d^2}$$

because the power is spread over the surface of a sphere. Intensity follows an inverse square law.

> **TRY IT**
>
> A star has luminosity 4.0×10^{26} W and is 2.0×10^{17} m away. Find the intensity at the Earth.

$$I = \frac{4.0 \times 10^{26}}{4\pi \times (2.0 \times 10^{17})^2} = 8.0 \times 10^{-10}\ \text{W m}^{-2}.$$

Note Academy

If the luminosity of a star is known (a standard candle), measuring I gives the distance: $d = \sqrt{\frac{L}{4\pi I}}$.

Measuring distances

Trigonometric parallax

As the Earth orbits the Sun, a nearby star appears to shift against the distant stars. The **parallax angle** p is half the total shift, measured against the Earth-Sun distance of 1 AU. For small angles in arcseconds, the distance in parsecs is

$$d = \frac{1}{p}$$

> **TRY IT**
>
> A star has a parallax of 0.25 arcsecond. Find its distance.

$$d = \frac{1}{0.25} = 4.0 \text{ parsecs.}$$

Parallax works for stars within a few hundred parsecs. Further away the angle is too small to measure, and standard candles are used instead.

The Hertzsprung-Russell diagram

An **HR diagram** plots stellar luminosity against surface temperature (or spectral class). The temperature axis is drawn **backwards**, with hot stars on the left.

- The **main sequence** runs from hot, luminous stars to cool, dim ones. Stars on it are fusing hydrogen to helium in their cores. Our Sun is here.
- **Red giants** are cool but very luminous, so they must be very large.
- **White dwarfs** are hot but dim, so they must be small.

The life cycle of a star

1. A cloud of gas collapses under gravity and heats up, forming a **protostar**.
2. When the core is hot enough, hydrogen fusion starts and it joins the **main sequence**, where it spends most of its life.
3. When the core hydrogen runs out, the star expands and cools into a **red giant**.
4. A star of about the Sun's mass sheds its outer layers and leaves a **white dwarf**.
5. A much more massive star ends in a **supernova**, leaving a neutron star or black hole.

Note Academy

On the HR diagram, a star moves from the main sequence to the red giant region and then, for a low-mass star, to the white dwarf region, as its core fuel runs out.

Redshift and the expanding universe

When a source of waves moves away from an observer, the received wavelength is stretched. For light this is a **redshift**. For speeds much less than c:

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{\Delta f}{f} \approx \frac{v}{c}$$

TRY IT

A spectral line of wavelength 656 nm is observed at 660 nm from a distant galaxy. Find the speed of recession.

$z = \dfrac{660 - 656}{656} = 6.1 \times 10^{-3}$, so $v = zc = 1.8 \times 10^{6}$ m s^{-1}.

Hubble's law says the recession speed of a galaxy is proportional to its distance: $v = H_0 d$. This shows the universe is expanding. Running the expansion backwards gives an age of about $1/H_0$, but the value of H_0 is uncertain, so the age and the ultimate fate of the universe, whether it expands forever or recollapses, are still debated.

Note Academy