Refraction and total internal reflection

Refractive index, Snell's law, the critical angle and total internal reflection, Huygens' construction for diffraction, and the pulse-echo technique.

Refraction

When a wave crosses the boundary between two media its speed changes. If it meets the boundary at an angle, its direction changes too. This is **refraction**. Frequency stays the same, so a change of speed means a change of wavelength.

Refractive index and Snell's law

The **refractive index** of a material is

$$n = \frac{c}{v}$$

where c is the speed of light in a vacuum and v the speed in the material. Air has $n \approx 1$, water about 1.33 and glass about 1.5.

At the interface between medium 1 and medium 2, **Snell's law** gives

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

with both angles measured from the **normal**.

> **TRY IT**
>
> Light in air strikes the surface of water ($n = 1.33$) at an angle of incidence of 40°. Find the angle of refraction.

$\sin\theta_2 = \dfrac{1 \times \sin 40°}{1.33} = 0.483$, so $\theta_2 = 28.9°$. The ray bends **towards** the normal in the denser medium.

Measuring the refractive index of a solid

Pass a ray through a rectangular block, measure the angle of incidence θ_1 and the angle of refraction θ_2 for several values, and plot $\sin\theta_1$ against $\sin\theta_2$. The gradient is n.

Total internal reflection

When light goes from a **denser** to a **less dense** medium it bends **away** from the normal. As the angle of incidence increases, the refracted ray moves towards the boundary. At the **critical angle** C the angle of

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refraction is 90°.

$$n_1 \sin C = n_2 \sin 90° \quad \Rightarrow \quad \sin C = \frac{n_2}{n_1} \quad \text{(for air outside: } \sin C = \frac{1}{n}\text{)}$$

For angles of incidence **greater than** C there is no refracted ray. All the light is reflected: **total internal reflection**.

> **Two conditions for total internal reflection: light goes from a more dense to a less dense medium, and the angle of incidence exceeds the critical angle.**

> **TRY IT**
>
> Find the critical angle for glass of refractive index 1.50 in air.

$\sin C = \dfrac{1}{1.50} = 0.667$, so $C = 41.8°$.

> **TRY IT**
>
> Light in this glass meets a glass-air surface at 50°. Will it emerge?

$50° > 41.8°$, so the light is totally internally reflected and does not emerge.

Uses

Optical fibres guide light by repeated total internal reflection along a core with a higher refractive index than its cladding. Prisms in binoculars and reflectors use it too.

Diffraction and Huygens' construction

Huygens' construction treats every point on a wavefront as a source of secondary wavelets. The new wavefront is the line tangent to the wavelets after a short time.

When a wave passes through a gap, the wavelets spread out into the region behind the edges. This is **diffraction**. It is greatest when the gap is about the same size as the wavelength.

> **NOTE**
>
> A wave with wavelength much smaller than the gap travels almost straight through with only slight spreading at the edges.

For a diffraction grating, $n\lambda = d \sin\theta$ gives the angle of the nth-order maximum for a grating of line spacing d.

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Pulse-echo

In the **pulse-echo** technique a short pulse of waves (ultrasound, radar, sonar) is sent out and the time t for the echo to return is measured. The distance to the reflecting object is

$$d = \frac{vt}{2}$$

because the pulse travels there and back. A string of pulses scanned across a body gives a map of the boundaries inside it.

TRY IT

An ultrasound pulse in tissue ($v = 1500$ m s^{-1}) returns after 60 μs. Find the depth of the boundary.

$$d = \frac{1500 \times 60 \times 10^{-6}}{2} = 0.045 \text{ m, or 4.5 cm.}$$

NOTE

The pulse must be short enough that the echo returns after the pulse has ended, and the wavelength small enough to show small detail.

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