# Particle accelerators and relativistic effects

Thermionic emission, linear accelerators and cyclotrons, the radius of a charged particle's path in a magnetic field, annihilation, energy units, and relativistic increase in lifetime.

## Thermionic emission and accelerating particles

### Thermionic emission

When a metal is heated, some of its conduction electrons gain enough energy to leave the surface. This is **thermionic emission**. An electron gun heats a filament to release electrons and then accelerates them with a potential difference $V$ between the filament and an anode.

An electron or ion of charge $q$ accelerated through a potential difference $V$ gains kinetic energy

$$E_k = qV$$

so a particle of mass $m$ reaches speed $v = \sqrt{\frac{2qV}{m}}$ if it is not moving at a speed near that of light.

> **TRY IT**
>
> Electrons are accelerated from rest through 2.0 kV. Find their speed. ($e = 1.60 \times 10^{-19}$ C, $m_e = 9.11 \times 10^{-31}$ kg.)

$$v = \sqrt{\frac{2 \times 1.60 \times 10^{-19} \times 2000}{9.11 \times 10^{-31}}} = 2.6 \times 10^{7} \text{ m s}^{-1}.$$

### Electric and magnetic fields in accelerators

A **linear accelerator (linac)** uses a line of metal drift tubes joined to an alternating supply. A particle is accelerated across each gap between tubes and travels at constant speed inside a tube, where there is no field. The tubes get longer along the machine so that the particle always reaches a gap just as the supply reverses.

A **cyclotron** has two D-shaped hollow electrodes ("dees") in a uniform magnetic field. The magnetic field makes the particles move in semicircles. They are accelerated by an alternating electric field each time they cross the gap between the dees, so they spiral outwards with larger radius at each half-turn.

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## The path of a charged particle in a magnetic field

A particle of charge $q$ moving at speed $v$ perpendicular to a magnetic field $B$ feels a force $F = Bqv$ at right angles to its velocity. This force is the centripetal force:

$$Bqv = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{Bq} = \frac{p}{Bq}$$

**The radius of the path is proportional to the momentum. Faster particles go round bigger circles, and a stronger field gives a tighter path.**

NOTE

The time for one revolution, $T = \dfrac{2\pi m}{Bq}$, does not depend on speed (for non-relativistic speeds). This is why a cyclotron can use a fixed-frequency supply.

# Particle interactions

In any interaction between particles, **charge**, **energy** and **momentum** are conserved.

## Annihilation

A particle and its **antiparticle** have the same mass and opposite charge. When they meet they **annihilate**: their mass is converted to energy, usually two photons going in opposite directions so that momentum is conserved.

$$e^- + e^+ \longrightarrow \gamma + \gamma$$

If the pair was nearly at rest, each photon has energy $m_e c^2 = 0.511$ MeV. The reverse process, **pair production**, makes a particle and its antiparticle from a single photon in the field of a nucleus. The photon needs at least the rest energy of both particles: $2m_e c^2 = 1.02$ MeV for an electron and a positron.

## Energy units

Particle energies are given in **electronvolts**: 1 eV $= 1.60 \times 10^{-19}$ J, with MeV $= 10^6$ eV and GeV $= 10^9$ eV. Using $E = mc^2$, masses are given as **MeV/c²** or GeV/c².

TRY IT

Express a proton mass of $1.67 \times 10^{-27}$ kg in MeV/c².

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$E = mc^2 = 1.67 \times 10^{-27} \times (3.00 \times 10^8)^2 = 1.50 \times 10^{-10}$ J $= \dfrac{1.50 \times 10^{-10}}{1.60 \times 10^{-13}} = 939$ MeV, so the mass is 939 MeV/c².

## High energies and the structure of matter

To investigate very small structures, the probing particles need a short wavelength. The de Broglie wavelength is

$$\lambda = \frac{h}{p}$$

so a smaller wavelength needs a larger momentum, and hence a larger energy. Resolving the structure inside a nucleon, about $10^{-15}$ m across, needs particles with energies of GeV. Accelerators also supply the energy needed to create new, massive particles.

## Relativistic effects

At speeds close to the speed of light $c$, Newton's mechanics does not give the right answer. One consequence is that a moving clock runs slower. A particle's lifetime $t$ in the laboratory frame is longer than its lifetime $t_0$ at rest:

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}} = \gamma t_0$$

> **TRY IT**
>
> A muon has a lifetime of 2.2 $\mu$s at rest. It is created high in the atmosphere and travels at $0.98c$. Find its lifetime as measured on Earth.

$\gamma = \dfrac{1}{\sqrt{1 - 0.98^2}} = 5.03$, so $t = 5.03 \times 2.2 = 11$ $\mu$s.

In 2.2 $\mu$s a muon at $0.98c$ would travel only about 650 m, but in 11 $\mu$s it travels about 3 km, so muons reach the ground. This is evidence for **time dilation**.

> **NOTE**
>
> The relativistic effect is only significant when $v$ is a sizeable fraction of $c$. At $0.1c$ the factor $\gamma$ is only 1.005.

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