Oscillations

Oscillation terms and angular frequency, the defining equation of simple harmonic motion, displacement, velocity and acceleration equations and graphs, energy interchange, damping, and resonance.

17.1 Simple harmonic oscillations

Terms

Term	Meaning
Displacement x	distance from the equilibrium position, in a given direction
Amplitude x_0	maximum displacement
Period T	time for one complete oscillation
Frequency f	oscillations per unit time, $f = 1/T$
Angular frequency ω	$\omega = 2\pi f = 2\pi/T$, in rad/s
Phase difference	how far one oscillation is ahead of another, in radians or degrees

The defining equation

Simple harmonic motion (SHM) occurs when the acceleration is proportional to the displacement from a fixed point, and is always in the opposite direction, so directed towards that point:

$$a = -\omega^2 x$$

The minus sign is what makes it SHM: the acceleration always points back towards the equilibrium. A graph of a against x is a straight line through the origin with gradient $-\omega^2$.

Examples include a mass on a spring and a simple pendulum at small angles.

Solutions

If timing starts at the equilibrium position:

$$x = x_0 \sin \omega t \qquad v = v_0 \cos \omega t$$

where $v_0 = \omega x_0$ is the maximum speed. If timing starts at the maximum displacement, swap to $x = x_0 \cos \omega t$.

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The speed at any displacement is:

$$v = \pm\omega\sqrt{x_0^2 - x^2}$$

Position	Displacement	Speed	Acceleration
Equilibrium	0	maximum, ωx_0	0
Extreme	$\pm x_0$	0	maximum, $\omega^2 x_0$

TRY IT

A mass oscillates with amplitude 0.040 m at 2.0 Hz. Find the maximum speed and maximum acceleration. $\omega = 2\pi \times 2.0 = 12.6$ rad/s. $v_0 = \omega x_0 = 0.50$ m/s. $a_0 = \omega^2 x_0 = 6.3$ m/s².

Graphs

- Velocity is the gradient of displacement against time, and it leads displacement by a quarter of a cycle ($\pi/2$).
- Acceleration is the gradient of velocity against time, and it is exactly opposite to the displacement (π out of phase).

17.2 Energy in simple harmonic motion

During SHM, energy continually changes between kinetic and potential, while the total stays constant, if there is no damping.

- At **equilibrium**: all kinetic.
- At the **extremes**: all potential.

$$E = \tfrac{1}{2}m\omega^2 x_0^2$$

This comes from the maximum kinetic energy, $\frac{1}{2}mv_0^2$, with $v_0 = \omega x_0$. So the energy is proportional to the amplitude squared.

The kinetic energy at displacement x is $\frac{1}{2}m\omega^2(x_0^2 - x^2)$, and the potential energy is $\frac{1}{2}m\omega^2 x^2$.

TRY IT

Double the amplitude of an SHM with the same frequency. What happens to the energy and the maximum speed? The energy goes up 4 times and the maximum speed goes up 2 times. The period does not change.

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17.3 Damped and forced oscillations, resonance

Damping

A resistive force acting on an oscillating system causes **damping**: it removes energy, so the amplitude falls over time.

Damping	What happens
Light	many oscillations, with the amplitude falling gradually (exponentially) and the period almost unchanged
Critical	returns to equilibrium in the shortest possible time, with no oscillation
Heavy	no oscillation, and a slow return to equilibrium, taking much longer than critical

Car suspensions are close to critically damped, so that the car settles quickly without bouncing.

Forced oscillations and resonance

A **free** oscillation happens at the system's **natural frequency**. In a **forced** oscillation, a periodic driving force makes the system oscillate at the driver's frequency.

Resonance is a maximum amplitude of oscillation, occurring when the driving frequency equals the system's natural frequency. Energy is then transferred from the driver most efficiently.

More damping makes the peak lower and broader, and it moves the peak slightly to a lower frequency.

NOTE

Exam tip: resonance can be useful (tuning a radio, magnetic resonance imaging) or harmful (a bridge shaking in the wind, or a building in an earthquake). When a question asks you to sketch, label the peak at f_0, and draw the more heavily damped curve below it and slightly to the left.

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