PHYSICS YPH11 · CHAPTER 12

Motion in a circle

The radian, angular displacement and angular speed, v = r omega, centripetal acceleration and centripetal force, and how to solve circular motion problems.

12.1 Kinematics of uniform circular motion

The radian

An angle in **radians** is arc length divided by radius. One **radian** is the angle at the centre of a circle subtended by an arc equal in length to the radius.

A full circle has arc length $2\pi r$, so a complete turn is 2π rad. So $360° = 2\pi$ rad, and 1 rad $\approx 57°$.

Angular speed

Angular speed ω is the angle swept out by the radius per unit time, in rad/s.

$$\omega = \frac{2\pi}{T} = 2\pi f$$

In time t, an object moves through an arc $s = r\theta$, so its linear speed is:

$$v = r\omega$$

> TRY IT
>
> **The Earth's radius is 6.4×10^6 m and it turns once in 24 hours. Find the speed of a point on the Equator.** $\omega = 2\pi/(24 \times 3600) = 7.3 \times 10^{-5}$ rad/s. $v = r\omega = 470$ m/s.

12.2 Centripetal acceleration

Why there is an acceleration

An object in a circle at constant speed has a constantly changing **direction**, so its velocity changes. It therefore accelerates, and there must be a resultant force.

- A force of constant size, always **perpendicular to the velocity**, changes the direction but not the speed.
- It points towards the centre of the circle, so we call it the **centripetal** force.
- It does no work, because it is at right angles to the motion, so the kinetic energy stays constant.

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The equations

$$a = r\omega^2 = \frac{v^2}{r}$$

$$F = mr\omega^2 = \frac{mv^2}{r}$$

Centripetal force is not an extra force. It is the resultant of the real forces on the object, such as tension, gravity, friction or a normal contact force. In your free-body diagram, never add a separate "centripetal force" arrow.

Solving problems

Identify which real forces act, find their resultant towards the centre, and set that equal to mv^2/r.

TRY IT

A 0.50 kg ball on a 1.0 m string moves in a vertical circle at 6.0 m/s at the top. Find the tension there. At the top, both the weight and tension act down towards the centre. $T + mg = mv^2/r$, so $T = 0.50 \times 36/1.0 - 0.50 \times 9.81 = 18 - 4.9 = 13$ N.

At the **bottom** of a vertical circle, tension acts up towards the centre and the weight acts away, so $T - mg = mv^2/r$. This makes the tension greatest at the bottom. At the top, the string goes slack if $v^2 < rg$.

Banked turns and conical pendulums

For a conical pendulum, or an aircraft banking, the force at an angle θ has two jobs:

- The vertical component balances the weight, $T\cos\theta = mg$.
- The horizontal component provides the centripetal force, $T\sin\theta = mv^2/r$.

Dividing gives $\tan\theta = v^2/(rg)$, which is independent of the mass.

NOTE

Exam tip: if you are given a period or a rotation rate, $mr\omega^2$ is usually easier. If you are given a speed, use mv^2/r. Always convert revolutions per minute into rad/s first, by multiplying by $2\pi/60$.

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