Kinematics

Distance, displacement, speed, velocity and acceleration, what the gradients and areas of motion graphs mean, the equations of uniformly accelerated motion, measuring g, and projectiles.

Equations of motion

Definitions

Quantity	Meaning	Type
Distance	total length of the path travelled	scalar
Displacement	straight-line distance from the start, in a stated direction	vector
Speed	distance moved per unit time	scalar
Velocity	rate of change of displacement	vector
Acceleration	rate of change of velocity	vector

A car going once round a circular track has travelled a large distance but has zero displacement, and its average velocity is zero.

Motion graphs

Graph	Gradient gives	Area under it gives
Displacement–time	velocity	(not useful)
Velocity–time	acceleration	displacement
Acceleration–time	(not useful)	change in velocity

For a curved displacement–time graph, the velocity at an instant is the gradient of the **tangent** at that point. Area below the time axis on a velocity–time graph counts as negative displacement.

Deriving the equations

These equations apply only when the acceleration is **constant** and the motion is in a straight line. Let u be the initial velocity, v the final velocity, a the acceleration, t the time and s the displacement.

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From the definition of acceleration, $a = (v - u)/t$, so:

$$v = u + at$$

The area under the velocity–time line is a trapezium, so $s = \frac{1}{2}(u + v)t$. Substituting for v and for t in turn gives:

$$s = ut + \tfrac{1}{2}at^2 \qquad v^2 = u^2 + 2as$$

> **Choose a positive direction first and stick to it. If up is positive, a ball thrown upwards has $u > 0$ and $a = -9.81$ m/s².**

> TRY IT
>
> **A stone is thrown straight up at 12 m/s. How high does it rise?** At the top, $v = 0$. $0 = 12^2 + 2(-9.81)s$, so $s = 144/19.62 = 7.3$ m.

Measuring the acceleration of free fall

Drop a steel ball through a measured height h, starting a timer when it is released and stopping it when it reaches a trapdoor or light gate at the bottom. From rest, $h = \frac{1}{2}gt^2$.

Repeat for several heights and plot h against t^2. This is a straight line through the origin with gradient $\frac{1}{2}g$, so $g = 2 \times \text{gradient}$. Using a graph averages out random errors, and an intercept reveals a systematic error such as a delay in releasing the ball.

Projectile motion

When an object moves with constant velocity in one direction and constant acceleration at right angles to it, **treat the two directions separately**. They share only the time.

For a projectile with no air resistance:

- **Horizontally:** no force, so the velocity is constant and $s_x = u_x t$.
- **Vertically:** acceleration g downwards, so use the equations of motion.

The path is a parabola. At the top, the vertical velocity is zero but the horizontal velocity is unchanged, so the speed is not zero there.

> TRY IT

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A ball rolls off a table 1.2 m high at 3.0 m/s. How far from the table does it land? Vertically, from rest: $1.2 = \frac{1}{2}(9.81)t^2$, so $t = 0.495$ s. Horizontally: $s = 3.0 \times 0.495 = 1.5$ m.

NOTE

Exam tip: to resolve an initial speed u at angle θ above the horizontal, use $u_x = u\cos\theta$ and $u_y = u\sin\theta$. The time of flight comes from the vertical motion, and the range from the horizontal motion.

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