Ideal gases

The mole and the Avogadro constant, the ideal gas equation in both forms, the Boltzmann constant, the assumptions of kinetic theory, deriving pressure from molecular motion, root-mean-square speed, and mean kinetic energy.

15.1 The mole

Amount of substance is an SI base quantity, with base unit the **mole** (mol). One mole of any substance contains $N_A = 6.02 \times 10^{23}$ particles, the **Avogadro constant**.

Number of particles: $N = nN_A$, where n is the amount in moles.

15.2 Equation of state

An **ideal gas** obeys $pV \propto T$ at all pressures, volumes and temperatures, where T is the thermodynamic temperature, in kelvin. Real gases behave like this at low pressures and at temperatures well above their boiling point.

$$pV = nRT$$

where $R = 8.31$ J K^{-1} mol^{-1} is the molar gas constant. In terms of the number of molecules, N:

$$pV = NkT$$

where $k = R/N_A = 1.38 \times 10^{-23}$ J/K is the **Boltzmann constant**.

Always use kelvin and SI units: pressure in Pa, volume in m³. A fixed mass of gas has pV/T constant, so $p_1V_1/T_1 = p_2V_2/T_2$.

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A gas at 20 °C and 1.0×10^5 Pa fills 0.024 m³. How many moles are there? $T = 293$ K. $n = pV/(RT) = (1.0 \times 10^5 \times 0.024)/(8.31 \times 293) = 0.99$ mol.

15.3 Kinetic theory of gases

Basic assumptions

1. A gas contains a very large number of molecules in **rapid, random motion**.
2. The volume of the molecules themselves is **negligible** compared with the volume of the gas.

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3. There are **no forces** between the molecules, except during collisions.
4. **Collisions are elastic**, and they take a negligible time compared with the time between collisions.

How molecules cause pressure

A molecule hits the wall and rebounds, so its momentum changes, and the wall exerts a force on it. By Newton's third law, it exerts an equal force on the wall. The very many molecular collisions add up to a steady pressure.

Deriving the pressure equation

Take a cube of side L containing N molecules, each of mass m. Consider one molecule with velocity c_x in the x direction.

1. Change of momentum at a wall: $2mc_x$.
2. Time between collisions with that wall: $2L/c_x$.
3. Average force on the wall: $F = 2mc_x/(2L/c_x) = mc_x^2/L$.
4. Pressure from all N molecules: $p = F/L^2 = Nm\langle c_x^2\rangle/L^3$. So $pV = Nm\langle c_x^2\rangle$.
5. In three dimensions, the motion is random, so $\langle c_x^2\rangle = \frac{1}{3}\langle c^2\rangle$.

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle$$

Here $\langle c^2\rangle$ is the **mean-square speed**: square the speeds, then average them.

The **root-mean-square speed** is $c_{\text{r.m.s.}} = \sqrt{\langle c^2\rangle}$. It is the square root of the mean of the squares, which is not the same as the mean speed.

> **TRY IT**
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> **Four molecules have speeds of 100, 200, 300 and 400 m/s. Find the r.m.s. speed.** Mean of squares: $(10^4 + 4\times10^4 + 9\times10^4 + 16\times10^4)/4 = 7.5\times10^4$. The r.m.s. speed is 274 m/s. The mean speed is only 250 m/s.

Mean kinetic energy and temperature

Equate the two expressions for pV: $\frac{1}{3}Nm\langle c^2\rangle = NkT$. So $m\langle c^2\rangle = 3kT$, and:

$$\tfrac{1}{2}m\langle c^2\rangle = \tfrac{3}{2}kT$$

> **The average translational kinetic energy of a molecule is proportional to the thermodynamic temperature, and it does not depend on the gas. Doubling the kelvin temperature doubles the**

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mean kinetic energy, and multiplies the r.m.s. speed by $\sqrt{2}$.

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Find the mean kinetic energy of a molecule at 300 K. $E = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.2 \times 10^{-21}$ J.

At the same temperature, lighter molecules have the same mean kinetic energy as heavier ones, so they move faster. Hydrogen molecules move much faster than oxygen molecules in the same container.

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