# Gravitational fields

Gravitational field strength and field lines, Newton's law of gravitation, circular orbits and geostationary satellites, g = GM/r squared, gravitational potential, and gravitational potential energy.

## 13.1 Gravitational field

A **gravitational field** is a field of force: a region where a mass feels a force. **Gravitational field strength** at a point is the force per unit mass:

$$g = \frac{F}{m}$$

The unit is N/kg, which is the same as m/s².

### Field lines

Field lines show the direction of the force on a small mass. Around a planet they point radially inwards, and they get further apart with distance, so the field weakens. Near the surface over a small region, the lines are close to parallel and equally spaced, so the field is nearly **uniform**.

## 13.2 Gravitational force between point masses

### Newton's law of gravitation

For a point outside a uniform sphere, the mass of the sphere may be treated as a **point mass at its centre**. Two point masses attract with:

$$F = \frac{Gm_1m_2}{r^2}$$

where $G = 6.67 \times 10^{-11}$ N m² kg⁻² and $r$ is the distance between their centres. It is an inverse square law, so doubling $r$ makes the force four times smaller.

### Circular orbits

For a satellite in a circular orbit, gravity provides the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

which gives $v^2 = GM/r$. The satellite's mass cancels, and closer orbits are faster. Using $v = 2\pi r/T$ gives:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

So $T^2 \propto r^3$. You should be able to derive this, not just quote it.

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**A satellite orbits at $r = 7.0 \times 10^6$ m from the centre of the Earth, whose mass is $6.0 \times 10^{24}$ kg. Find its speed.** $v = \sqrt{GM/r} = \sqrt{6.67 \times 10^{-11} \times 6.0 \times 10^{24}/7.0 \times 10^6} = 7.6 \times 10^3$ m/s.

### Geostationary orbits

A **geostationary** satellite stays above the same point on the Earth's surface. For this, it must:

- have an orbital period of **24 hours**,
- orbit from **west to east**, the same direction as the Earth's rotation,
- be directly above the **Equator**.

There is only one possible radius, about $4.2 \times 10^7$ m from the Earth's centre.

## 13.3 Gravitational field of a point mass

Put a small mass $m$ in the field of a point mass $M$. Since $F = GMm/r^2$ and $g = F/m$:

$$g = \frac{GM}{r^2}$$

### Why g is nearly constant near the surface

The Earth's radius is about 6400 km. Rising 1 km changes $r$ by a tiny fraction, so $r^2$ and therefore $g$ barely change. This is why we use $g = 9.81$ N/kg and $\Delta E_P = mg\Delta h$ for small height changes.

## 13.4 Gravitational potential

### Definition

The **gravitational potential** at a point is the work done per unit mass in bringing a small test mass from infinity to that point.

$$\phi = -\frac{GM}{r}$$

The unit is J/kg.

**Potential is always negative in a gravitational field. Gravity is attractive, so the field does work as a mass comes in from infinity, where the potential is defined to be zero. It takes energy input to lift a mass back out to infinity.**

Potential is a **scalar**, so potentials from several masses simply add.

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## Gravitational potential energy

The potential energy of a mass $m$ at a point is $m\phi$:

$$E_P = -\frac{GMm}{r}$$

The change in potential energy between two radii is:

$$\Delta E_P = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where $r_2 > r_1$ for a mass moving outwards, and the potential energy rises.

TRY IT

**Find the energy needed to move a 1000 kg satellite from the Earth's surface ($r = 6.4 \times 10^6$ m) to $r = 7.0 \times 10^6$ m, ignoring kinetic energy.** $\Delta E = GMm(1/r_1 - 1/r_2) = 6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 1000 \times (1.563 \times 10^{-7} - 1.429 \times 10^{-7}) = 5.4 \times 10^9$ J.

## Field and potential

The field strength is the negative of the potential gradient, $g = -\Delta\phi/\Delta r$. On a graph of $\phi$ against $r$, the gradient gives the size of $g$.

NOTE

Exam tip: gravity and electric fields use almost the same equations. Remember that the distance $r$ is always measured from the **centre**, not the surface, and that the potential's curve varies as $1/r$ while the field's curve varies as $1/r^2$.

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