Forces, density and pressure

Centre of gravity, moments, couples and torque, the principle of moments and equilibrium, vector triangles, density, pressure, hydrostatic pressure and upthrust.

Turning effects of forces

Centre of gravity

The **centre of gravity** of an object is the single point at which its whole weight may be taken to act. For a uniform beam it is at the middle.

The moment of a force

moment = force × perpendicular distance from the pivot to the line of action of the force

The unit is N m. If the force is at an angle, use the perpendicular distance, or use the component of the force perpendicular to the beam.

Couples and torque

A **couple** is a pair of equal and opposite forces whose lines of action are different. It produces rotation only, with no resultant force, so there is no acceleration of the object's centre.

torque of a couple = one of the forces × perpendicular distance between the forces

Equilibrium of forces

The principle of moments

> **For an object in equilibrium, the sum of the clockwise moments about any point equals the sum of the anticlockwise moments about that same point.**

Tip: take moments about the point where an unknown force acts. That force then has zero moment and disappears from the equation.

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Conditions for equilibrium

An object is in equilibrium when **both**:

1. the resultant force on it is zero, and
2. the resultant torque about any point is zero.

> TRY IT
>
> **A uniform 4.0 m beam of weight 200 N rests on supports at each end. A 300 N load sits 1.0 m from the left end. Find the force at the right support.** Moments about the left end: $300 \times 1.0 + 200 \times 2.0 = F \times 4.0$, so $F = 175$ N. The left support then carries $500 - 175 = 325$ N.

Vector triangles

If three coplanar forces keep an object in equilibrium, then drawn head to tail they form a **closed triangle**, because their resultant is zero. Their lines of action also meet at a single point, otherwise there would be a resultant torque.

Density and pressure

Definitions

Density is mass per unit volume, $\rho = m/V$, measured in kg m^{-3}. Water is about 1000 kg m^{-3}.

Pressure is the normal force per unit area, $p = F/A$, measured in pascals (1 Pa = 1 N m^{-2}).

Hydrostatic pressure

Consider a column of fluid of height Δh and cross-section A. Its volume is $A\Delta h$, its mass is $\rho A\Delta h$, and its weight is $\rho A\Delta h\, g$. Dividing this force by the area gives:

$$\Delta p = \rho g \Delta h$$

The pressure in a fluid depends on the depth and density, not on the shape of the container. To find the **total** pressure at a depth, add the atmospheric pressure at the surface.

> TRY IT
>
> **Find the total pressure 20 m under water.** $\Delta p = 1000 \times 9.81 \times 20 = 1.96 \times 10^5$ Pa. Adding atmospheric pressure, 1.01×10^5 Pa, gives 3.0×10^5 Pa.

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Upthrust

Pressure increases with depth, so the fluid pushes up harder on the bottom of a submerged object than it pushes down on the top. The difference is the upward force called **upthrust**.

For a cylinder of height h and end area A: upthrust $= \rho g h A = \rho g V$, where V is the volume displaced.

$$F = \rho g V$$

Archimedes' principle: the upthrust equals the weight of the fluid displaced. Here ρ is the density of the fluid, and V is the volume of fluid displaced. An object floats when the upthrust equals its weight.

TRY IT

A 0.60 kg block of volume 8.0×10^{-4} m³ is held under water. Find the resultant force on it when released. Upthrust $= 1000 \times 9.81 \times 8.0 \times 10^{-4} = 7.85$ N. Weight $= 5.89$ N. The resultant is 1.96 N upwards, so it rises.

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