Deformation of solids

Tensile and compressive forces, Hooke's law and the spring constant, stress, strain and the Young modulus and how to measure it, elastic and plastic behaviour, and the energy stored in a stretched material.

Stress and strain

Load and extension

Tensile forces stretch an object, and **compressive** forces squash it. The **load** is the force applied. The **extension** is the increase in length, and **compression** is the decrease in length.

Hooke's law

Up to the **limit of proportionality**, the extension is proportional to the load:

$$F = kx$$

Here k is the **spring constant** (N/m), the force per unit extension. A stiffer spring has a larger k.

For springs combined, $\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$ in series and $k = k_1 + k_2$ in parallel. In series each spring has the full load and the extensions add. In parallel the springs share the load.

Stress, strain and the Young modulus

The spring constant depends on the size of the sample. These quantities describe the **material** instead.

Quantity	Definition	Unit
Stress σ	force per unit cross-sectional area, $\sigma = F/A$	Pa
Strain ε	extension per unit original length, $\varepsilon = x/L$	none
Young modulus E	stress divided by strain, $E = \sigma/\varepsilon$	Pa

$$E = \frac{\sigma}{\varepsilon} = \frac{FL}{Ax}$$

The Young modulus applies within the limit of proportionality. It is the gradient of the straight part of a stress–strain graph. Steel is about 2×10^{11} Pa.

> **TRY IT**

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A steel wire 2.0 m long of area 1.0×10^{-6} m² carries 50 N. Using $E = 2.0 \times 10^{11}$ Pa, find the extension. $x = FL/(AE) = (50 \times 2.0)/(1.0 \times 10^{-6} \times 2.0 \times 10^{11}) = 5.0 \times 10^{-4}$ m, or 0.50 mm.

Measuring the Young modulus of a wire

1. Clamp a long thin wire (at least 2 m, so that the extension is measurable) and pass it over a pulley, with a marker on the wire above a fixed ruler.
2. Measure the original length L from the clamp to the marker, with a metre rule.
3. Measure the diameter d with a micrometer at several places along the wire and take the mean. Then $A = \pi d^2/4$.
4. Add masses in equal steps and record the extension each time.
5. Plot load F against extension x. The gradient of the straight part is F/x, so $E = \text{gradient} \times L/A$.

Wear eye protection, in case the wire snaps, and keep your feet clear of the masses.

Elastic and plastic behaviour

Elastic and plastic deformation

- **Elastic deformation:** the material returns to its original shape when the load is removed.
- **Plastic deformation:** the material is permanently deformed and does not return to its original shape.
- **Elastic limit:** the point beyond which the deformation becomes plastic. It is at or just beyond the limit of proportionality.

Work done and elastic potential energy

The work done in stretching a material is the **area under the force–extension graph**. Within the limit of proportionality the graph is a straight line through the origin, so the area is a triangle:

$$E_P = \tfrac{1}{2}Fx = \tfrac{1}{2}kx^2$$

This work is stored as elastic potential energy, and it can be recovered when the load is removed.

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A spring with $k = 200$ N/m is stretched by 0.10 m. How much energy is stored? What speed could it give a 0.50 kg trolley? $E_P = \tfrac{1}{2}(200)(0.10^2) = 1.0$ J. Then $1.0 = \tfrac{1}{2}(0.50)v^2$, so $v = 2.0$ m/s.

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Beyond the elastic limit, the unloading line runs parallel to the loading line but does not return to the origin. The area between the two curves is energy transferred to heat in the material and is not recovered, and the permanent extension remains.

NOTE

Exam tip: when a question says "doubling the load", check whether the wire stays within its limit of proportionality before using a proportion. Note also that $\frac{1}{2}Fx$ only applies to the straight part. For a curved graph, count squares.

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