Capacitance

Capacitance of isolated spheres and parallel plates, capacitors in series and parallel, energy stored from the area under a graph, and exponential discharge through a resistor with the time constant RC.

19.1 Capacitors and capacitance

Capacitance is the charge stored per unit potential:

$$C = \frac{Q}{V}$$

The unit is the **farad**, where 1 F = 1 C/V.

- For a **parallel plate capacitor**, it is the charge on one plate per unit p.d. between the plates.
- For an **isolated spherical conductor**, it is the charge on it per unit potential at its surface. Combining $C = Q/V$ with $V = Q/(4\pi\varepsilon_0 r)$ gives $C = 4\pi\varepsilon_0 r$.

A capacitor's plates hold equal and opposite charges. The net charge is zero, and Q means the charge on one plate.

Capacitors in series and parallel

In parallel, the p.d. is the same across each, and the charges add: $Q = Q_1 + Q_2$. Dividing by V gives:

$$C = C_1 + C_2 + \dots$$

In series, each capacitor holds the same charge, and the p.d.s add: $V = V_1 + V_2$. Dividing by Q gives:

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

> **These are the opposite way round from resistors. The series total is always smaller than the smallest capacitor.**

TRY IT

Capacitors of 3.0 μF and 6.0 μF are in series with a 12 V supply. Find the charge on each.
$1/C = 1/3 + 1/6$, so $C = 2.0$ μF. $Q = CV = 24$ μC, the same on each. The p.d.s are 8 V and 4 V.

19.2 Energy stored in a capacitor

Charging a capacitor means moving charge against an increasing p.d., so the work done is the **area under the potential–charge graph**, which is a triangle.

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$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = \frac{Q^2}{2C}$$

It is not QV, because the p.d. builds up from zero. The cell does transfer QV, and half of that goes to heat in the circuit's resistance, whatever the size of the resistance.

> TRY IT
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> **A 470 µF capacitor is charged to 12 V. How much energy does it store?** $W = \frac{1}{2} \times 470 \times 10^{-6} \times 12^2 = 0.034$ J.

19.3 Discharging a capacitor

When a capacitor discharges through a resistor, the current is large at first, because the p.d. is large. As the charge falls, the p.d. falls and the current falls. So the rate of loss of charge is proportional to the charge left, which gives an **exponential decay**.

$$x = x_0 e^{-t/RC}$$

where x can be the **charge, the p.d. or the current**.

The time constant

The **time constant** is:

$$\tau = RC$$

It is the time for the charge to fall to $1/e$, or about **37%**, of its starting value. After $5RC$ the capacitor is effectively discharged. A larger R makes the current smaller, and a larger C means there is more charge to remove, so both make discharge slower.

> TRY IT
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> **A 100 µF capacitor charged to 10 V discharges through 50 kΩ. Find the time constant and the p.d. after 10 s.** $\tau = 100 \times 10^{-6} \times 50 \times 10^3 = 5.0$ s. $V = 10e^{-10/5.0} = 10e^{-2} = 1.4$ V.

Finding the time constant from a graph

- From an exponential curve, read the time at which the value falls to 37%.
- Taking natural logs gives $\ln x = \ln x_0 - t/RC$. So a graph of $\ln V$ against t is a straight line with gradient $-1/RC$.

> NOTE

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Exam tip: time constants have units of seconds, because ohms × farads = seconds. Halving time is not the same as the time constant: it is $0.693RC$. Current in the circuit reverses direction on discharge compared with charging.

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