# Probability

Probability by counting, the addition and multiplication rules, exclusive and independent events, tree diagrams, and conditional probability.

## Probability

### Probability by counting

When all outcomes are equally likely:

$$\mathrm{P}(A) = \frac{\text{number of outcomes in } A}{\text{total number of outcomes}}$$

> **TRY IT**
>
> Two fair dice are thrown. Find the probability that the total is 9.

Of the 36 equally likely pairs, $(3, 6)$, $(4, 5)$, $(5, 4)$ and $(6, 3)$ give 9. So the probability is $\frac{4}{36} = \frac{1}{9}$.

A table of outcomes for two dice, or a list, avoids missing any.

### Using permutations and combinations

Counting methods give the numbers for the fraction.

> **TRY IT**
>
> 3 balls are taken from a bag of 4 red and 6 blue. Find the probability of exactly 2 red.

$$\frac{{}^{4}\mathrm{C}_{2} \times {}^{6}\mathrm{C}_{1}}{{}^{10}\mathrm{C}_{3}} = \frac{6 \times 6}{120} = 0.3$$

### The rules of probability

> $\mathrm{P}(A') = 1 - \mathrm{P}(A)$, **and** $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$A \cup B$ means "$A$ or $B$ or both". $A \cap B$ means "both $A$ and $B$". The overlap is subtracted, because it has been counted twice.

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## Exclusive and independent events

These two ideas sound alike but are different.

| Term | Meaning | Test |
|---|---|---|
| Mutually exclusive | Cannot both happen | $\mathrm{P}(A \cap B) = 0$, so $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$ |
| Independent | One happening does not change the chance of the other | $\mathrm{P}(A \cap B) = \mathrm{P}(A) \times \mathrm{P}(B)$ |

**TRY IT**

$\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.4$ and $\mathrm{P}(A \cap B) = 0.2$. Are $A$ and $B$ independent? Are they exclusive? Find $\mathrm{P}(A \cup B)$.

$0.5 \times 0.4 = 0.2$, so they are independent. $\mathrm{P}(A \cap B) \neq 0$, so they are not exclusive.

$\mathrm{P}(A \cup B) = 0.5 + 0.4 - 0.2 = 0.7$.

**NOTE**

Independence must be shown by the calculation with numbers. Saying "one doesn't affect the other" earns no marks in a test like this. Two events with non-zero probabilities cannot be both exclusive and independent.

## Tree diagrams

Multiply along the branches, and add between the paths that fit. Without replacement, the numbers on later branches change.

**TRY IT**

A bag has 5 red and 3 blue balls. Two are taken without replacement. Find the probability that they are the same colour.

$\mathrm{P}(\mathrm{RR}) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$ $\quad$ $\mathrm{P}(\mathrm{BB}) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$

$\mathrm{P}(\text{same}) = \frac{26}{56} = \frac{13}{28}$

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## Conditional probability

$\mathrm{P}(A \mid B)$ means the probability of $A$, given that $B$ has happened.

$$\mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$$

The word "given", or wording such as "if it is known that", signals this.

**TRY IT**

In the bag example, given that the balls are the same colour, find the probability that both are red.

$\mathrm{P}(\mathrm{RR} \mid \text{same}) = \frac{\frac{20}{56}}{\frac{26}{56}} = \frac{10}{13}$

Conditional probability lets you test for independence another way: $A$ and $B$ are independent if $\mathrm{P}(A \mid B) = \mathrm{P}(A)$.

## Reversing a tree

Questions often give the later branches and ask about an earlier one.

**TRY IT**

30% of days are rainy. A bus is late on 40% of rainy days, and on 10% of other days. The bus is late. Find the probability it is raining.

$\mathrm{P}(\text{late}) = 0.3 \times 0.4 + 0.7 \times 0.1 = 0.12 + 0.07 = 0.19.$

$\mathrm{P}(\text{rain} \mid \text{late}) = \frac{0.12}{0.19} = 0.632$

## Venn diagrams

Venn diagrams help when two or three events overlap. Fill in from the centre outwards, and check the regions total 1.

**TRY IT**

20 students study French, 15 study Spanish and 5 study both, out of 40. Find the probability that a random student studies neither.

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French or Spanish: $20 + 15 - 5 = 30$. So 10 study neither, and the probability is $\frac{10}{40} = 0.25$.

**NOTE**

For "at least one", the quickest way is usually $1 - \mathrm{P}(\text{none})$.

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