Newton's laws of motion

Newton's second law, mass and weight, motion on slopes and in lifts, and connected particles joined by strings and rods.

Newton’s laws of motion

The three laws

Law	In brief
First	A particle stays at rest or moves at constant velocity unless a resultant force acts on it
Second	Resultant force $= ma$, in the direction of the acceleration
Third	If A pushes B, then B pushes A with an equal and opposite force

$F = ma$, where F is the resultant force. Forces in the direction of the acceleration are positive. Mass is in kg and acceleration is in m s^{-2}.

Weight and mass. Mass is the amount of matter, in kg. Weight is the force of gravity on it, in newtons: $W = mg$.

TRY IT

A car of mass 1200 kg has a driving force of 3000 N and a resistance of 600 N. Find its acceleration.

$3000 - 600 = 1200a \quad \Rightarrow \quad a = 2 \text{ m s}^{-2}$

Lifts and vertical motion

TRY IT

A person of mass 70 kg stands in a lift accelerating upwards at 2 m s^{-2}. Find the reaction from the floor.

Taking up as positive: $R - 700 = 70 \times 2$, so $R = 840$ N.

Note Academy

The person feels heavier when the lift accelerates upwards, and lighter when it accelerates downwards. This also applies when a lift going up slows down, since the acceleration is then downwards.

Inclined planes

Resolve along the slope for $F = ma$, and perpendicular to the slope for R, where there is no acceleration.

> **TRY IT**
>
> A 5 kg block slides down a rough plane at 30°, with $\mu = 0.2$. Find its acceleration.

Perpendicular: $R = 50\cos 30° = 43.3$ N. So $F = 0.2 \times 43.3 = 8.66$ N.

Along the slope: $50\sin 30° - 8.66 = 5a$, so $a = 3.27$ m s^{-2}.

> **NOTE**
>
> Mass cancels here, so the acceleration does not depend on it. On a plane going up, friction points down the slope. When the block stops, ask whether it stays: it slides back only if $mg\sin\theta > \mu R$.

Connected particles

For two particles joined by a light inextensible string or a rod:

- They have the same size of acceleration while the string is taut.
- The tension is the same along a string over a smooth pulley.

> **Write $F = ma$ for each particle separately, then solve the pair. Or treat the system as one object to find the acceleration, then use one particle to find the tension.**

Particles over a pulley

> **TRY IT**
>
> Particles of 3 kg and 2 kg hang from a light string over a smooth pulley. Find the acceleration and the tension.

For the 3 kg particle, moving down: $30 - T = 3a$. For the 2 kg particle, moving up: $T - 20 = 2a$.

Adding gives $10 = 5a$, so $a = 2$ m s^{-2}. Then $T = 24$ N.

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A 4 kg particle on a rough table, with $\mu = 0.25$, is joined by a string over the edge to a hanging 2 kg particle. Find the acceleration.

On the table, $R = 40$ N, so $F = 10$ N and $T - 10 = 4a$.

The hanging particle has $20 - T = 2a$.

Adding gives $10 = 6a$, so $a = \frac{5}{3}$ m s^{-2} and $T = 16.7$ N.

Vehicles and tow bars

A car and trailer are connected particles with the same acceleration. The tow bar pulls the trailer forward and the car backward.

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A 1000 kg car tows a 200 kg trailer. The driving force is 3000 N, and the resistances are 500 N on the car and 100 N on the trailer. Find the acceleration and the tension.

The whole system: $3000 - 600 = 1200a$, so $a = 2$ m s^{-2}.

The trailer alone: $T - 100 = 200 \times 2$, so $T = 500$ N.

NOTE

When braking, the tow bar can be in **thrust**, pushing the trailer back on the car. Take the direction as you expect, and a negative answer means the force is the other way.

After the string goes slack

If a hanging particle hits the ground, the string goes slack and the two parts move separately:

1. Find the common acceleration and the speed when it hits the ground.
2. The other particle continues with this speed, with a new acceleration: only friction on a table, or only gravity if it is hanging.
3. Find the further distance it moves, and add this if a total is asked for.

TRY IT

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In the table example above, the 2 kg particle falls 1 m and hits the ground. How much further does the 4 kg particle travel?

Speed at impact: $v^2 = 2 \times \frac{5}{3} \times 1 = \frac{10}{3}$.

After this, friction alone acts on the table: $-10 = 4a$, so $a = -2.5$ m s^{-2}.

$0 = \frac{10}{3} + 2(-2.5)s \quad \Rightarrow \quad s = \frac{2}{3}$ m

A new stage of motion needs a new calculation of the acceleration. The value from the first stage does not carry over.

Note Academy