Momentum

Linear momentum as a vector, conservation of momentum in a direct impact, particles that coalesce or rebound, and collisions in a chain.

Momentum

Linear momentum

Momentum $= mv$. It is a vector, with the same direction as the velocity. Its unit is kg m s^{-1}, or equivalently N s.

Because momentum has a direction, a velocity in the opposite direction gives a negative momentum.

TRY IT

A 2 kg ball moving at 5 m s^{-1} hits a wall and rebounds at 3 m s^{-1}. Find the change in momentum.

Taking the original direction as positive, the momentum goes from 10 to -6 kg m s^{-1}. The change has size 16 kg m s^{-1}. It is not $10 - 6$.

Conservation of momentum

When two objects collide, the forces between them are equal and opposite and last for the same time. So the total momentum does not change, provided no external force acts along the line of motion.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Draw a before-and-after sketch, pick a positive direction, and give any velocity moving the other way a negative sign.

Particles that coalesce

Coalesce means they join together, and move on as one combined particle.

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A 3 kg particle moving at 4 m s^{-1} strikes a stationary 2 kg particle, and they coalesce. Find their common speed.

$3 \times 4 + 2 \times 0 = 5v \quad \Rightarrow \quad v = 2.4 \text{ m s}^{-1}$

Particles moving towards each other

TRY IT

Particles A (2 kg) and B (3 kg) move towards each other at 5 m s^{-1} and 2 m s^{-1} respectively. After the collision, A moves back at 1 m s^{-1}. Find the velocity of B.

Taking A's original direction as positive:

$2(5) + 3(-2) = 2(-1) + 3v \quad \Rightarrow \quad 4 = -2 + 3v \quad \Rightarrow \quad v = 2$

B moves at 2 m s^{-1} in A's original direction, so B's direction has been reversed too.

NOTE

If you leave a velocity as v with a positive direction and get a negative answer, the particle moves the other way. State its direction in words.

Kinetic energy in a collision

Momentum is conserved in these problems, but kinetic energy usually is not: some of it becomes heat and sound.

TRY IT

Find the loss of kinetic energy in the collision above.

Before: $\frac{1}{2}(2)(5^2) + \frac{1}{2}(3)(2^2) = 25 + 6 = 31$ J.

After: $\frac{1}{2}(2)(1^2) + \frac{1}{2}(3)(2^2) = 1 + 6 = 7$ J.

The loss is 24 J.

Chains of collisions

Treat each collision on its own, using the velocity after one as the velocity before the next.

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A (1 kg) moves at 6 m s^{-1} towards B (2 kg), which is at rest. After the collision, A is at rest. B then strikes C (2 kg), also at rest, and they coalesce. Find the final speed.

First collision: $1 \times 6 = 2v_B$, so $v_B = 3$ m s^{-1}.

Second collision: $2 \times 3 = 4v$, so $v = 1.5$ m s^{-1}.

To see whether a further collision happens, compare velocities. A later collision needs the one behind to be moving faster than the one in front.

Joining with other topics

Momentum questions often come with other mechanics:

- **Kinematics:** find the speed just before the collision using the constant acceleration formulae, and use the speed afterwards for the rest of the motion.
- **Energy:** find the speed at the bottom of a slope or track, collide, then find how far it travels.
- **Friction:** the combined particle slows down under friction after impact. Recalculate R and F with the new mass.

NOTE

Collisions in this syllabus are direct impacts in one straight line. No further rule is needed for rebounds: the question will give enough information.

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