Kinematics

Distance, speed, displacement, velocity and acceleration, motion graphs, the constant acceleration formulae, and using calculus when the acceleration varies.

Kinematics of motion in a straight line

Scalars and vectors

Scalar (size only)	Vector (size and direction)
Distance	Displacement
Speed	Velocity
Time	Acceleration

A particle moving 5 m forward and 3 m back has travelled a distance of 8 m but has a displacement of 2 m. Average speed uses total distance, and average velocity uses displacement.

Motion graphs

Graph	Gradient	Area under it
Displacement–time	Velocity	Not used
Velocity–time	Acceleration	Displacement

On a velocity–time graph, area above the axis is displacement forward and area below it is displacement backward. For total distance, add the sizes of all the areas.

TRY IT

A train speeds up uniformly from rest to 20 m s^{-1} in 40 s, travels at this speed for 200 s, then slows uniformly to rest in 60 s. Find the distance.

The graph is a trapezium with parallel sides of 200 s and 300 s, and height 20:

$$\text{distance} = \tfrac{1}{2}(200 + 300) \times 20 = 5000 \text{ m}$$

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The constant acceleration formulae

These are valid only when the acceleration is constant and the motion is in a straight line.

$$v = u + at \qquad s = ut + \tfrac{1}{2}at^2 \qquad s = \tfrac{1}{2}(u + v)t \qquad v^2 = u^2 + 2as \qquad s = vt - \tfrac{1}{2}at^2$$

List the quantities you know and want, then pick the formula that contains them.

Formula	Missing quantity
$v = u + at$	s
$s = ut + \tfrac{1}{2}at^2$	v
$s = \tfrac{1}{2}(u + v)t$	a
$v^2 = u^2 + 2as$	t
$s = vt - \tfrac{1}{2}at^2$	u

Motion under gravity

An object moving freely in a vertical line has acceleration $g = 10$ m s^{-2} downwards. Choose a positive direction and keep to it.

> **TRY IT**
>
> A stone is thrown vertically upwards at 20 m s^{-1}. Find its greatest height and the time until it returns.

Taking up as positive, $a = -10$. At the top, $v = 0$:

$$0 = 20^2 + 2(-10)s \quad \Rightarrow \quad s = 20 \text{ m}$$

It returns when $s = 0$: $0 = 20t - 5t^2$, so $t = 4$ s.

> **TRY IT**
>
> Find when the stone is 15 m above the point of projection.

$15 = 20t - 5t^2$, so $t^2 - 4t + 3 = 0$ and $t = 1$ s or $t = 3$ s. There are two answers, one going up and one coming down.

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Two particles

When particles meet or one catches the other, write each one's displacement using the same clock, then set up the condition that links them.

> **TRY IT**
>
> Car A passes a point at a constant 15 m s^{-1}. At that moment, car B starts from rest there, accelerating at 2 m s^{-2}. When does B catch A?

$15t = \frac{1}{2}(2)t^2$, so $t^2 = 15t$ and $t = 15$ s.

> **NOTE**
>
> If one particle starts later, its time is $t - T$. Set this up at the start rather than patching it later.

Variable acceleration

When the acceleration changes, the formulae above do not apply. Use calculus.

$v = \dfrac{\mathrm{d}s}{\mathrm{d}t}$ and $a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$. Going back: $v = \int a\,\mathrm{d}t$ and $s = \int v\,\mathrm{d}t$.

You know	To get	Do this
Displacement	Velocity	Differentiate
Velocity	Acceleration	Differentiate
Acceleration	Velocity	Integrate and find the constant
Velocity	Displacement	Integrate and find the constant

> **TRY IT**
>
> A particle starts from O with velocity $v = 3t^2 - 12t + 9$ m s^{-1}. Find when it is at rest, its acceleration then, and the total distance in the first 3 seconds.

At rest: $3(t-1)(t-3) = 0$, so $t = 1$ or $t = 3$. Its acceleration is $a = 6t - 12$, which is -6 m s^{-2} at $t = 1$ and 6 m s^{-2} at $t = 3$.

Its displacement is $s = t^3 - 6t^2 + 9t$, with no constant, since $s = 0$ at $t = 0$.

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$s(1) = 4$ and $s(3) = 0$. It moves 4 m forward, then 4 m back, so the distance is 8 m even though the displacement is 0.

A particle reverses when $v = 0$. For total distance, find those times and add the sizes of each stage's displacement. A single definite integral across a reversal gives the displacement.

Stages of motion

Many questions join two or more stages: a car accelerating, then cruising, then braking. The final velocity of one stage is the initial velocity of the next. A velocity–time sketch makes these questions easier, even when not asked for.

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