Forces and equilibrium

Identifying forces, components and resultants, equilibrium of a particle, the normal contact force and friction, limiting equilibrium, and Newton's third law.

Forces and equilibrium

Forces to look for

Start every mechanics question with a clear diagram showing each force on the object.

Force	Acts
Weight, $W = mg$	Vertically downwards. Take $g = 10$ m s^{-2} unless told otherwise
Normal reaction, R	At right angles to the surface, away from it
Friction, F	Along the surface, opposing motion or the tendency to move
Tension, T	Along a string, away from the object
Thrust	Along a rod, towards the object

Components and resultants

A force F at angle θ to a direction has a component $F\cos\theta$ along it and $F\sin\theta$ at right angles to it.

To combine forces, find the total X and the total Y:

Resultant $= \sqrt{X^2 + Y^2}$, at an angle $\tan^{-1}\dfrac{Y}{X}$ to the direction of X.

TRY IT

A 10 N force acts at 30° above the horizontal, and a 6 N force acts horizontally in the same sense. Find the resultant.

$X = 10\cos 30° + 6 = 14.66$ and $Y = 10\sin 30° = 5$.

The resultant is $\sqrt{14.66^2 + 5^2} = 15.5$ N, at $\tan^{-1}\dfrac{5}{14.66} = 18.8°$ above the horizontal.

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Equilibrium of a particle

A particle is in equilibrium when the resultant force is zero: the sum of the components in any direction is zero.

Resolve in two directions at right angles. Choose directions that leave the fewest unknowns: along and perpendicular to a slope, or horizontally and vertically on level ground.

TRY IT

A particle of weight 20 N hangs from two strings at 30° and 60° to the vertical. Find the tensions.

Resolving horizontally: $T_1 \sin 30° = T_2 \sin 60°$.

Resolving vertically: $T_1 \cos 30° + T_2 \cos 60° = 20$.

The first gives $T_1 = \sqrt{3}\, T_2$. Then $\frac{3}{2} T_2 + \frac{1}{2} T_2 = 20$, so $T_2 = 10$ N and $T_1 = 17.3$ N.

NOTE

With three forces in equilibrium, a closed triangle of forces and the sine rule is a useful alternative to resolving.

Contact forces and friction

A contact force between two surfaces has two components: the normal force R and the friction force F.

A **smooth** contact has no friction. It is a model: real surfaces are never exactly smooth, but friction may be small enough to ignore.

For a **rough** contact, friction opposes the direction the object would otherwise move, and it grows only as large as it needs to be, up to a limit:

$F \leq \mu R$, where μ is the coefficient of friction. At the point of slipping, or while sliding, $F = \mu R$.

Limiting equilibrium means the object is on the point of moving, so $F = \mu R$.

NOTE

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$F = \mu R$ holds only when limiting or sliding. A block at rest with a small push has less friction than this. Find F by resolving first, then check whether $F \leq \mu R$.

Inclined planes

On a slope at angle θ, resolve along and perpendicular to the slope:

down the slope: $mg \sin\theta$ into the slope: $mg \cos\theta$

TRY IT

A block rests on a rough plane inclined at 30°, on the point of sliding down. Find μ.

Perpendicular: $R = mg \cos 30°$. Along: $F = mg \sin 30°$. With $F = \mu R$:

$$\mu = \frac{\sin 30°}{\cos 30°} = \tan 30° = 0.577$$

A force at an angle

A pull at an angle lifts part of the weight off the surface, which reduces R and so reduces the friction. A push downwards at an angle does the opposite.

TRY IT

A 4 kg block on a rough horizontal surface, with $\mu = 0.5$, is pulled by a force P at 30° above the horizontal. It is about to move. Find P.

Vertically: $R + P \sin 30° = 40$, so $R = 40 - 0.5P$.

Horizontally: $P \cos 30° = F = 0.5R = 20 - 0.25P$.

So $1.116P = 20$, and $P = 17.9$ N.

Never assume $R = mg$. Find R by resolving, every time.

Friction either way

An object on a slope held by a force P may be about to slide down or about to be pushed up. Friction reverses between the two, so these give the least and greatest values of P:

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About to	Friction acts	Along the slope
Slide down	Up the slope	$P + F = mg \sin\theta$
Move up	Down the slope	$P = mg \sin\theta + F$

Newton's third law

When two objects touch, the force each exerts on the other is equal in size and opposite in direction. These two forces act on different objects, so they never cancel in one object's equation.

> **TRY IT**
>
> A book rests on a table. Name the partner of the book's weight.

The weight is the Earth's pull on the book. Its partner is the book's pull on the Earth. The normal reaction on the book is a different force, and it is paired with the book's push on the table.

Note Academy