Coordinate geometry

Straight lines in all their forms, parallel and perpendicular lines, the equation of a circle, tangents, and where lines and curves meet.

Coordinate geometry

Straight lines

For the points (x_1, y_1) and (x_2, y_2):

gradient $m = \frac{y_2 - y_1}{x_2 - x_1}$ midpoint $= (\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$ distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

A line can be written in three forms:

Form	Use it when
$y = mx + c$	You want the gradient and y-intercept
$y - y_1 = m(x - x_1)$	You know the gradient and one point
$ax + by + c = 0$	The question asks for it, usually with integer coefficients

TRY IT

Find the equation of the line through $(1, 2)$ and $(3, 8)$.

$m = \dfrac{8 - 2}{3 - 1} = 3$, so $y - 2 = 3(x - 1)$, which is $y = 3x - 1$.

Parallel and perpendicular lines

Parallel lines have equal gradients. Perpendicular lines have gradients with $m_1 m_2 = -1$, so the second gradient is $-\dfrac{1}{m_1}$.

TRY IT

Find the perpendicular bisector of the line joining $(1, 2)$ and $(3, 8)$.

Note Academy

The midpoint is $(2, 5)$ and the gradient of the line is 3, so the perpendicular gradient is $-\frac{1}{3}$:

$$y - 5 = -\tfrac{1}{3}(x - 2) \quad \Rightarrow \quad x + 3y = 17$$

The equation of a circle

A circle with centre (a, b) and radius r has equation

$$(x - a)^2 + (y - b)^2 = r^2$$

This is Pythagoras' theorem for the distance from the centre to any point (x, y) on the circle.

If the equation is expanded, complete the square in x and in y to get back to this form.

> **TRY IT**
>
> Find the centre and radius of $x^2 + y^2 - 6x + 4y - 12 = 0$.

$$(x - 3)^2 - 9 + (y + 2)^2 - 4 - 12 = 0 \quad \Rightarrow \quad (x - 3)^2 + (y + 2)^2 = 25$$

The centre is $(3, -2)$ and the radius is 5.

Circle facts to use

These come from circle geometry and turn a circle question into a straight-line question.

Fact	How it is used
A tangent is perpendicular to the radius at the point of contact	Find the radius gradient, then take the negative reciprocal
The perpendicular bisector of a chord passes through the centre	Two such bisectors meet at the centre
The angle in a semicircle is a right angle	If AB is a diameter and C is on the circle, AC is perpendicular to BC

Tangents to a circle

> **TRY IT**
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> Show that $(6, 2)$ lies on $(x - 3)^2 + (y + 2)^2 = 25$ and find the tangent there.

Note Academy

$3^2 + 4^2 = 25$, so the point is on the circle. The radius from $(3, -2)$ to $(6, 2)$ has gradient $\frac{4}{3}$, so the tangent has gradient $-\frac{3}{4}$:

$$y - 2 = -\tfrac{3}{4}(x - 6) \quad \Rightarrow \quad 3x + 4y = 26$$

Lines and circles

Substitute the line into the circle to get a quadratic. Its discriminant says how they meet: two points if positive, a tangent if zero, no meeting if negative.

> **TRY IT**
>
> Find the values of k for which $y = 2x + k$ is a tangent to $x^2 + y^2 = 5$.

$$x^2 + (2x + k)^2 = 5 \quad \Rightarrow \quad 5x^2 + 4kx + k^2 - 5 = 0$$

For a tangent, $16k^2 - 20(k^2 - 5) = 0$, so $100 - 4k^2 = 0$ and $k = \pm 5$.

Finding a circle from given facts

To write down a circle you need its centre and radius.

> **TRY IT**
>
> $A(1, 3)$ and $B(7, 11)$ are the ends of a diameter. Find the equation of the circle.

The centre is the midpoint, $(4, 7)$. The radius is the distance from the centre to A: $\sqrt{3^2 + 4^2} = 5$. So

$$(x - 4)^2 + (y - 7)^2 = 25$$

For a circle through three given points, find the perpendicular bisectors of two of the chords. They cross at the centre.

Points of intersection of graphs

The points where two graphs meet are the solutions of their equations taken together.

- Set the two expressions for y equal to find the x-coordinates.
- A repeated root means the graphs touch rather than cross.
- A question may ask you to use a graph to solve an equation: the roots of $\mathrm{f}(x) = \mathrm{g}(x)$ are the x-coordinates where $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$ cross.

> **NOTE**

Note Academy

When a question asks for an exact answer, or says "do not use a calculator", leave surds in. A rounded decimal loses the accuracy mark.

Note Academy