Circular measure

Radians, converting to and from degrees, arc length, sector area, and how to find the area of a segment.

Circular measure

What a radian is

One **radian** is the angle at the centre of a circle made by an arc equal in length to the radius. A whole circle has circumference $2\pi r$, so it contains 2π radians.

π radians $= 180°$

To convert degrees to radians, multiply by $\frac{\pi}{180}$. To convert radians to degrees, multiply by $\frac{180}{\pi}$.

Degrees	30°	45°	60°	90°	120°	180°	270°	360°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π	$\frac{3\pi}{2}$	2π

NOTE

Check your calculator's mode. Set it to radians when the question uses radians, including every time you calculate $\sin\theta$ for the area of a segment.

Arc length and sector area

For a sector of radius r and angle θ **in radians**:

Arc length $s = r\theta$. Sector area $A = \frac{1}{2}r^2\theta$.

Both come from taking a fraction $\frac{\theta}{2\pi}$ of the whole circle.

TRY IT

A sector has radius 6 cm and angle 1.2 radians. Find its arc length, area and perimeter.

$s = 6 \times 1.2 = 7.2$ cm $\qquad A = \frac{1}{2} \times 36 \times 1.2 = 21.6$ cm^2

Note Academy

The perimeter includes the two radii: $6 + 6 + 7.2 = 19.2$ cm.

Segments and chords

A **chord** joins two points on the circle. A **segment** is the region between a chord and its arc: the sector with the triangle removed.

The triangle has two sides of length r with angle θ between them, so its area is $\frac{1}{2}r^2 \sin\theta$. Therefore:

> **Area of segment** $= \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta = \frac{1}{2}r^2(\theta - \sin\theta)$

Cutting the triangle in half gives two right-angled triangles, which give the chord length:

$$\text{chord} = 2r \sin\tfrac{\theta}{2}$$

> **TRY IT**
>
> For the same sector, with $r = 6$ and $\theta = 1.2$, find the chord length and the area of the segment.

$$\text{chord} = 12 \sin 0.6 = 6.78 \text{ cm}$$

$$\text{segment} = \tfrac{1}{2} \times 36 \times (1.2 - \sin 1.2) = 18 \times 0.26796 = 4.82 \text{ cm}^2$$

Tackling harder questions

Most questions combine sectors with triangles, tangents or several circles. These habits help:

- **Mark every right angle.** A tangent makes a right angle with the radius, so a tangent question usually has a right-angled triangle in it, and $\tan\theta$ gives the tangent length.
- **Express the area or perimeter as a sum or difference** of sectors, triangles and straight lengths before calculating.
- **Find the missing angle using trigonometry** with a right-angled triangle, or the cosine rule with a triangle of known sides. The answer must be in radians before you use it in $r\theta$.
- **Keep exact values** when asked. With $\theta = \dfrac{\pi}{3}$, the area of the triangle uses $\sin\dfrac{\pi}{3} = \dfrac{\sqrt{3}}{2}$.

> **TRY IT**
>
> From a point T, tangents TA and TB touch a circle of radius 5 cm and centre O. Angle $AOB = \dfrac{2\pi}{3}$. Find the exact area between the tangents and the minor arc AB.

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Angle $AOT = \frac{\pi}{3}$ and angle OAT is a right angle, so $TA = 5\tan\frac{\pi}{3} = 5\sqrt{3}$.

The kite $OATB$ is two right-angled triangles, each of area $\frac{1}{2} \times 5 \times 5\sqrt{3}$, so its area is $25\sqrt{3}$.

The sector has area $\frac{1}{2} \times 25 \times \frac{2\pi}{3} = \frac{25\pi}{3}$.

$\text{Area} = 25\sqrt{3} - \frac{25\pi}{3}$

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