Trigonometry

Pythagoras' theorem, sine, cosine and tangent in right-angled triangles, and the sine rule, cosine rule and area rule for any triangle.

Pythagoras' theorem

$a^2 + b^2 = c^2$, where c is the hypotenuse (the longest side, opposite the right angle).

NOTE

This is only applicable to right-angled triangles.

TRY IT

Find the length of the missing side of this right-angled triangle.

$3^2 + 3^2 = c^2 \quad \Rightarrow \quad c = \sqrt{18} = 4.2 \text{ cm}$

Right-angled triangles

For an angle θ in a right-angled triangle: the **hypotenuse** is the longest side, the **opposite** side is the side facing θ, and the **adjacent** side is the side next to θ (that is not the hypotenuse).

Ratio	Formula
Sin	$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}}$
Cos	$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}}$
Tan	$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$

Memorise SOHCAHTOA: Sin = Opp/Hyp, Cos = Adj/Hyp, Tan = Opp/Adj.

NOTE

Note Academy

These are only applicable to right-angled triangles. You can use these formulae to find the third, missing thing if you are given two things. Answers should be written in degrees, with decimals correct to one decimal place.

TRY IT

Find the missing angle.

Since we have the lengths of the opposite side and the hypotenuse, we use sin:

$$\sin(\theta) = \frac{3}{5} \quad \Rightarrow \quad \theta = \sin^{-1}\left(\frac{3}{5}\right) = 36.9°$$

The shortest distance from a point to a line is the perpendicular distance.

TRY IT

Find the shortest distance between the point and the line.

Since the shortest distance is the perpendicular distance, we calculate the side opposite the angle using sine:

$$\sin(30°) = \frac{x}{5} \quad \Rightarrow \quad x = 5\sin(30°) = 2.5 \text{ cm}$$

TRY IT

A ramp rises 1 m over a distance of 4 m. Find the angle of elevation.

$$\tan(\theta) = \frac{1}{4} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{1}{4}\right) = 14.0°$$

Write the answer to 1 decimal place even if it is $.0$.

Non-right-angled triangles

Sine rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, **where a, b, c are sides and A, B, C are the angles opposite those sides.**

TRY IT

Note Academy

Find A.

The 4 cm side is opposite the 40° angle, and the 2 cm side is opposite A:

$$\frac{4}{\sin 40^\circ} = \frac{2}{\sin A} \quad \Rightarrow \quad \sin A = \frac{2 \sin 40^\circ}{4} \quad \Rightarrow \quad A = 18.7^\circ$$

NOTE

Keep the full calculator value until the end. Rounding $\frac{4}{\sin 40^\circ}$ to 6.22 early gives 18.8° instead.

Cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Use the cosine rule when you know:

- all three sides (SSS), or
- two sides and the angle between them (SAS).

TRY IT

Find the length of side A.

$$A^2 = 5^2 + 3^2 - 2(5)(3)\cos(80^\circ) = 34 - 30\cos 80^\circ$$

$$A = 5.37 \text{ cm}$$

Area rule

$$\text{Area} = \frac{1}{2}ab \sin C$$

Use it when you know two sides and the angle between them (SAS).

TRY IT

Find the area of the triangle below.

$$\text{Area} = \tfrac{1}{2}(5)(3)\sin 80^\circ = 7.4 \text{ cm}^2$$

Note Academy

Exact trigonometric values

NOTE

Extended only. This topic is in the Extended syllabus, so it is asked in Papers 2 and 4. If you sit the Core papers you do not need it.

You must know these values **without a calculator**:

x	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	not defined

NOTE

A memory trick: the sine values go up as $\frac{\sqrt{0}}{2}, \frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, \frac{\sqrt{4}}{2}$. The cosine values are the same list backwards. And $\tan x = \frac{\sin x}{\cos x}$.

TRY IT

Find the exact length x in a right-angled triangle where the hypotenuse is 10 cm and the angle opposite x is $30°$.

$x = 10 \sin 30° = 10 \times \frac{1}{2} = 5$ cm.

Trigonometric functions

NOTE

Extended only. This topic is in the Extended syllabus, so it is asked in Papers 2 and 4. If you sit the Core papers you do not need it.

Note Academy

The graphs, for $0° \leq x \leq 360°$

- $y = \sin x$ starts at 0, rises to a maximum of 1 at $90°$, is 0 at $180°$, falls to -1 at $270°$ and is 0 at $360°$.
- $y = \cos x$ starts at 1, is 0 at $90°$, is -1 at $180°$, 0 at $270°$ and 1 at $360°$. It is the sine graph shifted $90°$ to the left.
- $y = \tan x$ is 0 at $0°$, $180°$ and $360°$, rises steeply towards $90°$ and $270°$, where it is undefined (the graph has **asymptotes**).

Sine and cosine **repeat every** $360°$ and stay between -1 and 1. Tangent repeats every $180°$.

Solving trigonometric equations

A calculator gives **one** answer. Use the graph's symmetry to find the other one in $0° \leq x \leq 360°$.

Equation type	Second solution
$\sin x = k$	$180° - x$
$\cos x = k$	$360° - x$
$\tan x = k$	$180° + x$

First get the trigonometric ratio on its own (for example $2 \sin x = 1$ becomes $\sin x = \frac{1}{2}$), find the calculator angle, then use the graph to find the other solution in the range.

TRY IT

Solve $2 \sin x = 1$ for $0° \leq x \leq 360°$.

$\sin x = 0.5$, so $x = 30°$ from the calculator. The other solution is $180° - 30° = 150°$. So $x = 30°$ or $150°$.

TRY IT

Solve $\cos x + 1 = 0$ for $0° \leq x \leq 360°$.

$\cos x = -1$, which happens only at $x = 180°$.

Note Academy