Probability

The probability scale, calculating probabilities, relative and expected frequency, and combined events using sample space diagrams, Venn diagrams and tree diagrams, with and without replacement.

Introduction to probability

Probability tells us how **likely something is to happen**. It's like a measure of chance.

The **probability scale** is a number line from 0 to 1 where we can place the probability of events occurring.

0	0.25	0.5	0.75	1
Impossible	Unlikely	Maybe	Likely	Definite

We use the letter P to show probability.

- $P(A)$ means "the probability of event A happening".
- $P(A')$ means "the probability of A not happening".

For example, if $P(A) = 0.3$, then $P(A') = 0.7$.

Probability can be written in many ways (such as in the table).

Format	Example
Fraction	$\frac{1}{4}$
Decimal	0.25
Percentage	25%

How to calculate basic probability

Here is a general formula for calculating probability:

$$\text{Probability} = \frac{\text{number of desired outcomes}}{\text{number of total outcomes}}$$

TRY IT

What's the probability of rolling a 4 on a dice?

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- Total outcomes: 6 (a dice has the numbers 1 to 6).
- Desired outcome: 1 (only one 4).

$P(4) = \frac{1}{6}$

> **TRY IT**
>
> A bag has 3 red, 2 blue and 5 green balls. Find $P(\text{red})$.

- Total: $3 + 2 + 5 = 10$.
- Red $= 3$.

$P(\text{red}) = \frac{3}{10}$

> **The probability of a thing not happening $= 1-$ the probability of the thing happening:** $P(A') = 1 - P(A)$**.**

> **TRY IT**
>
> Find $P(\text{blue}')$ when $P(\text{blue}) = 0.2$.

$P(\text{blue}') = 1 - 0.2 = 0.8$

Relative and expected frequencies

Sometimes we can't calculate exact probabilities, so we do experiments instead and use the results to estimate. That's where **relative frequency** comes in.

$$\text{Relative frequency} = \frac{\text{number of times the event happened}}{\text{number of trials}}$$

> **TRY IT**
>
> You spin a spinner 100 times and get these results: red 25, blue 30, green 45. Estimate the probability of getting each colour.

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Colour	Spins
Red	25
Blue	30
Green	45

$P(\text{red}) = \frac{25}{100} = 0.25$ $P(\text{blue}) = \frac{30}{100} = 0.30$ $P(\text{green}) = \frac{45}{100} = 0.45$

These are estimates, not exact probabilities. **The more trials you do, the closer it gets to the real probability.**

- This spinner is not **fair**.
- A fair spinner would be one that has equal probability for all outcomes.
- In this example, the spinner would be fair if the probability of each outcome was $\frac{1}{3}$ (0.33).
- Since this spinner is not fair, it is called a **biased** spinner.

Once you know the probability of something happening, you can use it to predict how often it should happen in future trials. This is called the **expected frequency**.

> $\text{Expected frequency} = \text{probability} \times \text{number of trials}$

> **TRY IT**
>
> The probability of rolling a 6 is $\frac{1}{6}$. You roll the dice 60 times. How many 6s do you expect?

$\text{Expected frequency} = \frac{1}{6} \times 60 = 10$

Sometimes, outcomes can be **random**, meaning each outcome has a chance and there is no pattern. An expected frequency is therefore an average, not a promise: you may roll more or fewer than 10 sixes.

Probability of combined events

A combined event is when two or more things happen, like flipping two coins, or picking two balls one after another.

Sample space diagrams

> **TRY IT**

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A fair spinner with numbers 1 to 6 is spun two times. Find the probability that the sum of the two numbers is 7.

For this question, we make a **sample space diagram**, listing every possible outcome. The shaded squares are the sums that equal 7.

$$P(7) = \frac{\text{number of desired outcomes}}{\text{total}} = \frac{6}{36} = \frac{1}{6}$$

Venn diagrams

TRY IT

20 people were asked whether they play football, cricket or hockey. The Venn diagram shows the results. Find the probability that a person picked at random (a) plays football, and (b) plays cricket and hockey.

Check the total first: $6 + 2 + 1 + 1 + 5 + 3 + 2 = 20$ people.

(a) Everyone inside the football circle plays football: $6 + 2 + 1 + 1 = 10$.

$$P(\text{football}) = \frac{10}{20} = 0.5$$

(b) Cricket and hockey is the overlap of those two circles, which includes the centre: $2 + 1 = 3$.

$$P(\text{cricket and hockey}) = \frac{3}{20} = 0.15$$

NOTE

"Plays football" means everyone in the football circle, not only the region with football alone. Count every number that sits inside the circle (or circles) the question names.

Tree diagrams

TRY IT

On any day in January, the probability that the temperature at a weather station is above 14°C is 0.35. The temperature on two consecutive days in January is recorded. (a) Complete the tree diagram.

The probabilities on each pair of branches must add up to 1. The first day below 14°C has probability $1 - 0.35 = 0.65$. The temperature on the second day does not depend on the first day, so its branches have the same probabilities: 0.35 and 0.65.

Note Academy

TRY IT

(b) Find the probability that the temperature is above 14°C on both days.

To find the combined probabilities, **we multiply the probabilities of the events occurring** along the branches.

$$P(\text{above } 14°\text{C on both days}) = 0.35 \times 0.35 = 0.1225$$

Without replacement

TRY IT

A bag contains 12 balls. There are x black balls in the bag and the other balls are white. Two balls are taken at random from the bag without replacement. (a) Complete the tree diagram.

Without replacement means that the ball that was selected will not be put back with the other balls for the next selection. So for the second ball:

- the denominator reduces by 1, because there are only 11 balls left;
- if a black ball was taken first, there are $x - 1$ black balls left and still $12 - x$ white balls;
- if a white ball was taken first, there are still x black balls left and $12 - x - 1 = 11 - x$ white balls.

TRY IT

(b) Find an expression for the probability of taking one ball of each colour. Write your answer as a single fraction in terms of x.

There are 2 possibilities here: we take the black ball and then the white, or the white and then the black. Hence, we **add** the two probabilities.

$$\frac{x}{12} \times \frac{12-x}{11} + \frac{12-x}{12} \times \frac{x}{11} = \frac{x(12-x)}{132} + \frac{x(12-x)}{132} = \frac{2x(12-x)}{132} = \frac{x(12-x)}{66}$$

Along a branch, multiply. Between different routes to the same result, add.

Conditional probability

NOTE

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Extended only. This topic is in the Extended syllabus, so it is asked in Papers 2 and 4. If you sit the Core papers you do not need it.

A **conditional probability** is the probability of an event **given that** another event has already happened. It is written $P(A \mid B)$, "the probability of A given B".

When there is no replacement, the second probability changes. Use the numbers that are left.

From a tree diagram

Multiply along the branches, and use the changed numbers on the second set of branches.

TRY IT

A bag has 4 red and 6 blue counters. Two are taken without replacement. Find the probability that both are red.

First red: $\frac{4}{10}$. Then, with 3 red left among 9 counters, second red: $\frac{3}{9}$. So $P(\text{both red}) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15}$.

From a Venn diagram or a table

Restrict yourself to the group you are **given** and count within it.

$$P(A \mid B) = \frac{\text{number in both } A \text{ and } B}{\text{number in } B}$$

TRY IT

In a class of 30, 18 study French, 12 study Spanish and 5 study both. Find the probability that a student chosen at random studies French, given that they study Spanish.

Within the 12 Spanish students, 5 also study French: $P = \frac{5}{12}$.

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