Mensuration

Units of measure and conversions, perimeter and area, circles, sectors and arcs, and the surface area and volume of solids, including compound shapes, similar solids and frustums.

Units of measure

TRY IT

Convert $20\ \text{m}^2$ to cm^2.

To convert m to cm, we multiply by 100. But because we have a power of 2 with the units, we multiply by $100^2 = 10\,000$:

$$20 \times 10\,000 = 200\,000\ \text{cm}^2$$

The same principle is used for other questions. For example, to convert cm^3 to m^3, we divide by $100^3 = 1\,000\,000$.

$1\ \text{cm}^3 = 1\ \text{ml}$. **Hence** $1\ \text{L} = 1000\ \text{cm}^3$.

Area and perimeter

Perimeter is the sum of the lengths of all the sides of the shape.

TRY IT

Find the perimeter of the shape below.

$P = 3 + 5 + 5 + 6 = 19$ cm. Always remember to write the units.

TRY IT

The perimeter of the shape below is 14 cm. Find the missing length.

Perimeter = sum of all sides, so $14 = 4 + 3 + 2 + x$. Then $x = 14 - 9 = 5$ cm.

Note Academy

Area

Shape	Area formula	Example
Rectangle	$A = \text{length} \times \text{width} = LW$	$L = 4, W = 3$: $A = 4 \times 3 = 12\ \text{cm}^2$
Triangle	$A = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}bh$	$b = 3, h = 4$: $A = \frac{1}{2}(3)(4) = 6\ \text{cm}^2$
Parallelogram	$A = \text{base} \times \text{height} = bh$	$b = 3, h = 4$: $A = 4 \times 3 = 12\ \text{cm}^2$
Trapezium	$A = \frac{1}{2}(a+b) \times \text{height} = \frac{1}{2}(a+b)h$	$a = 2, b = 3, h = 4$: $A = \frac{1}{2}(2+3)(4) = 10\ \text{cm}^2$

NOTE

Unfortunately, you will have to memorise these formulae. Also make sure that you are choosing the correct dimensions to put into your formulae. For example, people use the slant height of the parallelogram rather than the vertical height. **Height is always $90°$ to the base.**

TRY IT

Find the area of the shape below.

- Area of A $= 4 \times 4 = 16\ \text{cm}^2$
- Area of B $= 8 \times 4 = 32\ \text{cm}^2$
- Area of C $= \frac{1}{2} \times 4 \times 5 = 10\ \text{cm}^2$
- Total area $= 16 + 32 + 10 = 58\ \text{cm}^2$

Circles, sectors and arcs

Thing	Formula	Example ($r = 3$ cm)
Circumference (C)	$C = 2\pi r$ or $C = \pi d$	$C = 2\pi(3) = 18.8$ cm
Area of a circle (A)	$A = \pi r^2$	$A = \pi(3)^2 = 28.3\ \text{cm}^2$
Arc length	$\text{Arc} = \frac{\theta}{360°} \times 2\pi r$	$\theta = 45°$: $\frac{45}{360}(2\pi \times 3) = 2.36$ cm
Area of a sector	$\text{Sector} = \frac{\theta}{360°} \times \pi r^2$	$\theta = 45°$: $\frac{45}{360}(\pi \times 3^2) = 3.53\ \text{cm}^2$

All answers are given to 3 significant figures.

Note Academy

Surface area and volume

You were introduced to various 3D shapes in the previous chapter. Now we will learn how to calculate the surface area and volume of those shapes.

The formulae in the table below will be given in the exam. However, it is preferable to memorise and understand them.

Shape	Surface area	Volume
Cuboid	$2(lw + lh + wh)$	$l \times w \times h$
Prism	Add all the face areas	Area of cross-section $\times$ length
Cylinder	$2\pi rh$ (curved face) $+2\pi r^2$	$\pi r^2 h$
Cone	πrl (curved face) $+\pi r^2$	$\frac{1}{3}\pi r^2 h$
Sphere	$4\pi r^2$	$\frac{4}{3}\pi r^3$
Pyramid	Add the base + all the triangular sides	$\frac{1}{3} \times$ base area $\times$ height

In the cone, l is the slant height.

> **TRY IT**
>
> Find the volume and surface area of the prism below.

Volume = area of face $\times$ length:

$$V = (\tfrac{1}{2} \times 2.6 \times 3) \times 10 = 39 \text{ cm}^3$$

Surface area = areas of all faces = 2 triangles + 3 rectangles:

$$SA = 2\,(\tfrac{1}{2} \times 2.6 \times 3) + 3(10 \times 3) = 7.8 + 90 = 97.8 \text{ cm}^2$$

> **NOTE**
>
> The three rectangles are each 10×3 here because the triangle's three sides are all about 3 cm. In general, use each rectangle's own width (the length of that side of the triangle).

A compound solid

> **TRY IT**

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Find the volume and the surface area of this solid.

Volume = volume of cone + volume of hemisphere. The hemisphere is half a sphere, so its volume is $\frac{1}{2} \times \frac{4}{3}\pi r^3$.

$$V = \tfrac{1}{3}\pi(4.5)^2(7.6) + \tfrac{1}{2} \times \tfrac{4}{3}\pi(4.5)^3 = 161.2 + 190.9 = 352 \text{ cm}^3$$

Surface area = curved surface of the cone + curved surface of the hemisphere. Find the slant height l with Pythagoras' theorem:

$$l = \sqrt{4.5^2 + 7.6^2} = \sqrt{78.01} = 8.83 \text{ cm}$$

The curved surface of the hemisphere is half of $4\pi r^2$, which is $2\pi r^2$:

$$SA = \pi(4.5)(8.83) + 2\pi(4.5)^2 = 124.9 + 127.2 = 252 \text{ cm}^2$$

Similar solids

TRY IT

Two geometrically similar jugs have volumes of 1000 cm^3 and 512 cm^3. They have circular bases. The diameter of the larger jug is 9 cm. Find the diameter of the smaller jug.

Volumes scale with the cube of the lengths:

$$\frac{V_1}{V_2} = \left(\frac{L_1}{L_2}\right)^3 \quad \Rightarrow \quad \frac{1000}{512} = \left(\frac{9}{x}\right)^3$$

Take the cube root of both sides: $\frac{10}{8} = \frac{9}{x}$, so $x = 7.2$ cm.

A harder surface area

TRY IT

The solid below has a square top of side 110 and a square base of side 80, and its sloping edges are 70 long. Find its surface area (in the same units squared).

For questions with lots of faces, label the faces and calculate the area of each separately. The top and bottom faces are squares. Each of the four sides is a trapezium with parallel sides 110 and 80 and sloping edge 70.

Make a trapezium separately and find its height. The base is wider than the top by $110 - 80 = 30$, so each end sticks out by 15:

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$h = \sqrt{70^2 - 15^2} = \sqrt{4675} = 68.4$

Area of one trapezium $= \frac{1}{2}(110 + 80)(68.4) = 6496$

$SA = 110^2 + 80^2 + 4 \times 6496 = 12\,100 + 6400 + 25\,984 \approx 44\,500$ square units

A frustum

TRY IT

The frustum below was made by cutting the point off a cone of height 8 cm, exactly half way down, so the frustum is half the height of the original cone. The radius of its top is 4 cm. Find the surface area and volume of the frustum.

By using similarity, we can find the radius of the smaller (missing) cone. It is half the height, so it is half as wide:

$\frac{8}{4} = \frac{4}{x} \quad \Rightarrow \quad x = 2$ cm

Now we find the volume and surface area of both cones and subtract them to find the frustum.

$V_1 = \frac{1}{3}\pi(4)^2(8) = 134 \text{ cm}^3 \qquad V_2 = \frac{1}{3}\pi(2)^2(4) = 16.8 \text{ cm}^3$

Volume of frustum $= 134 - 16.8 = 117 \text{ cm}^3$

For the curved surface, πrl with $l = \sqrt{r^2 + h^2}$:

$SA_1 = \pi \times 4 \times \sqrt{4^2 + 8^2} = 112 \text{ cm}^2 \qquad SA_2 = \pi \times 2 \times \sqrt{2^2 + 4^2} = 28 \text{ cm}^2$

The surface of the frustum is the curved part (the difference) plus the top circle and the bottom circle:

$SA = (112 - 28) + \pi(4)^2 + \pi(2)^2 = 84 + 50.3 + 12.6 = 147 \text{ cm}^2$

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