Geometry

The vocabulary of shapes, triangles, quadrilaterals, polygons, solids and circles, constructing triangles, nets, similarity, symmetry, and the angle and circle theorems.

Important terms

Term	Meaning	Idea
Vertex	A corner where two lines meet.	A corner of a triangle.
Plane	A flat surface that goes on forever.	A giant sheet of paper that never ends.
Parallel	Lines that never touch, no matter how far they go.	Train tracks.
Perpendicular bisector	A line that cuts another line in half at $90°$.	Cutting a straw in half straight across.
Bearing	The direction from one point to another, measured in degrees from North (clockwise).	North is $000°$.
Right angle	A $90°$ angle.	The corner of a square.
Acute angle	Less than $90°$ (small and sharp).	
Obtuse angle	Bigger than $90°$, but less than $180°$.	
Reflex angle	Bigger than $180°$, like a big loop.	
Interior angle	Angles inside a shape.	
Exterior angle	Angles outside the shape when the sides are extended.	
Similar	Same shape but different size.	A photo and its enlargement.
Congruent	Same shape and same size.	Two identical triangles.

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Triangles

Type	What it means
Equilateral	All sides and angles are equal (each angle is $60°$).
Isosceles	Two sides (and two angles) are equal.
Scalene	All sides and angles are different.
Right-angled	Has one $90°$ angle.

Quadrilaterals (four-sided shapes)

Shape	What it means
Square	All sides equal, all angles $90°$.
Rectangle	Opposite sides equal, all angles $90°$.
Kite	Two pairs of equal-length sides next to each other.
Rhombus	All sides equal, opposite angles equal.
Parallelogram	Opposite sides parallel and equal.
Trapezium	Only one pair of sides is parallel.

Polygons (many-sided shapes)

All the exterior angles of a polygon add up to $360°$.

- Exterior angle $= 180° -$ interior angle.
- Exterior angle $= 360° \div$ number of sides (regular polygons only).
- Sum of interior angles $= (n - 2) \times 180°$.
- Interior angle $= \dfrac{(n - 2) \times 180°}{n}$ (regular polygons only).

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Name	Sides
Regular polygon	All sides and angles equal.
Irregular polygon	Sides and angles not all the same.
Pentagon	5 sides
Hexagon	6 sides
Octagon	8 sides
Decagon	10 sides

TRY IT

Find the size of one interior angle of a regular hexagon.

$n = 6$, so the interior angles add up to $(6 - 2) \times 180° = 720°$. One angle is $720° \div 6 = 120°$. Check: the exterior angle is $360° \div 6 = 60°$, and $180° - 60° = 120°$.

Solids (3D shapes)

Name	Description
Cube	All faces are squares.
Cuboid	Like a box, with rectangle faces.
Prism	The same shape on both ends, with straight sides.
Cylinder	Like a can, with circular ends.
Pyramid	A flat base with triangular sides.
Cone	Like an ice cream cone: a round base and a point.
Sphere	Like a ball, with no edges or corners.
Hemisphere	Half a sphere.
Frustum	A chopped-off cone or pyramid.

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Circles

Term	Meaning
Centre	The middle point of a circle.
Radius	From the centre to the edge.
Diameter	Across the whole circle, through the centre.
Circumference	The distance around the circle.
Semicircle	Half of a circle.
Chord	A straight line across the circle (not through the centre).
Tangent	A line that touches the circle at exactly one point.
Major arc	A big curved part of the circle.
Minor arc	A small curved part of the circle.
Sector	A "pizza slice" part of a circle.
Segment	A part of the circle between a chord and the arc.

Constructing triangles using a compass

If you are asked to make a triangle with sides of specific lengths, follow these steps. You need paper, a compass with a pencil and a ruler.

> **TRY IT**
>
> Draw a triangle that has sides measuring 5 cm, 4 cm and 3 cm.

1. Draw a 5 cm line using a ruler.
2. Open your compass to 4 cm. Put the point on one end of the line and draw an arc above the line.
3. Open your compass to 3 cm. Put the point on the other end of the line and draw another arc that crosses the first one.
4. Draw two lines from the ends of the 5 cm line to the point where the two arcs cross. Measure the sides at the end to confirm that they are the lengths you need.

> **NOTE**
>
> Never rub out the arcs. They are the working and earn the marks.

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Nets

Nets are 2D shapes joined together that fold to make a 3D shape. Think of a cardboard box, such as a box that contains biscuits. Think of the net as a flat piece of cardboard before folding. After folding the net, it forms the 3D shape it is meant to.

TRY IT

What type of shape will these nets form?

NOTE

Hint: to solve this question, try to mentally fold the nets in your head. Otherwise, you should recall what solid has the shapes in the net.

A square with a triangle on each side folds into a **pyramid** (a square base and sides of triangles).

Two triangles joined by three rectangles fold into a **triangular prism**.

TRY IT

Draw the net of this triangular prism. Use a scale of 1 cm to represent 2 cm.

The prism has a right-angled triangle at each end, with sides 6, 8 and 10, and it is 12 long.

Scale is used to adjust drawings. Here the drawing would be too big, so we make it smaller by halving all the lengths. The net has five faces:

Face	Real size	Size in your drawing
Rectangle (the 6 side)	6×12	3×6 cm
Rectangle (the 8 side)	8×12	4×6 cm
Rectangle (the 10 side)	10×12	5×6 cm
Two triangles	base 8, height 6	base 4 cm, height 3 cm

Draw the three rectangles side by side, sharing their 6 cm sides, and attach one triangle to each end of the 4×6 rectangle (along its 4 cm sides).

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Similarity

When two shapes are similar, they have the same shape but different sizes. Their angles are equal, and their sides are in the same ratio.

The scale factor here is 2, because all the sides doubled in length.

When two shapes (like cubes or spheres) are similar and the lengths increase by scale factor k:

- Lengths increase by k.
- Areas increase by k^2.
- Volumes increase by k^3.

$$\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{\text{Length}_1}{\text{Length}_2}\right)^2 \textbf{ and } \frac{\text{Volume}_1}{\text{Volume}_2} = \left(\frac{\text{Length}_1}{\text{Length}_2}\right)^3$$

For corresponding sides: $\dfrac{\text{Side of } A_1}{\text{Side of } B_1} = \dfrac{\text{Side of } A_2}{\text{Side of } B_2}$

TRY IT

The two shapes below are similar. Find the missing length.

The 2 cm base edge matches the 6 cm base edge, and the 4 cm edge matches the missing length x:

$$\frac{6}{2} = \frac{x}{4} \quad \Rightarrow \quad x = 12 \text{ cm}$$

TRY IT

Two similar cones have heights 5 cm and 10 cm. The smaller one has volume 40 cm³. Find the volume of the larger one.

The length scale factor is $\frac{10}{5} = 2$, so the volume scale factor is $2^3 = 8$. The larger volume is $40 \times 8 = 320$ cm³.

Symmetry

Line symmetry (also called mirror symmetry): a shape has line symmetry if you can fold it and both halves match perfectly.

Rotational symmetry: a shape has rotational symmetry if it looks the same when you turn it (by less than a full circle). The number of times it looks the same in one full turn is the **order** of rotational symmetry. A

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shape that only looks the same after a full turn has order 1 (no rotational symmetry).

- The equilateral triangle has 3 lines of symmetry and rotational symmetry of order 3.
- The kite has 1 line of symmetry and rotational symmetry of order 1.

For all regular polygons, the number of sides equals both the number of lines of symmetry and the order of rotational symmetry.

TRY IT

Shade two more squares to make a shape with rotational symmetry of order 2.

Here is a 4×4 grid. A square is shaded (■) where it is grey in the original exercise: row 1, columns 3 and 4; row 3, column 3; and row 4, column 1.

	Col 1	Col 2	Col 3	Col 4
Row 1			■	■
Row 2				
Row 3			■	
Row 4	■			

Rotational symmetry of order 2 means the shape looks the same after a half turn $(180°)$ about the centre of the grid. A half turn sends the square in row r, column c to row $5 - r$, column $5 - c$. So:

- row 1, column 3 goes to row 4, column 2, which is not shaded yet;
- row 1, column 4 goes to row 4, column 1, which is shaded;
- row 3, column 3 goes to row 2, column 2, which is not shaded yet;
- row 4, column 1 goes to row 1, column 4, which is shaded.

So shade **row 2, column 2** and **row 4, column 2**.

NOTE

Tip: choose a line or a centre first, then shade the squares that are its matches.

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Angles

Angle rules at a point, on a straight line and between lines

Angles around a point add up to $360°$.

$w + x + y + z = 360°$

Angles on a straight line add up to $180°$.

$x + y + z = 180°$

Vertically opposite angles are equal.

Angles in parallel lines

Corresponding angles (F-shaped) are equal. Arrows on lines mean that the lines are parallel.

Alternate angles (Z-shaped) are equal. Also, the angles between the parallel lines on the same side (co-interior angles) add up to $180°$.

$a = b, \quad x = y, \quad a + x = 180°, \quad y + b = 180°$

Circle properties

Angle in a semicircle $= 90°$

When you make a triangle by connecting the ends of a diameter to a point on the circle, it will always be a right-angled triangle.

Angle between a tangent and a radius $= 90°$

The angle that a tangent makes with the radius at the point of contact will always be $90°$.

Angle at the centre $= 2\times$ angle at the circumference

The angle from a chord at the centre is twice that at the circumference of the circle.

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Angles in the same segment are equal

Similar to a semicircle, all the angles drawn from the same chord on the same side of it will be equal.

Opposite angles in a cyclic quadrilateral add to $180°$

If all the corners of a quadrilateral touch the circumference of a circle, the angles on opposite corners add up to $180°$.

$a + b = 180°$ $x + y = 180°$

Alternate segment theorem

When the corner of a triangle touches the tangent of a circle, the angle between the tangent and a chord equals the angle in the alternate (opposite) segment.

$x = y$ $a = b$

Equal chords are equidistant from the centre

If two chords in a circle are the same length, they are the same distance from the centre.

The perpendicular bisector of a chord passes through the centre

If you draw a perpendicular line that cuts a chord exactly in half, it will go through the centre of the circle.

Tangents from an external point are equal in length

If you draw two tangents from the same point outside a circle, they will both be the same length.

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