Coordinate Geometry

The length and midpoint of a line, the equation of a straight line, and parallel and perpendicular lines, including the perpendicular bisector.

Length and midpoint

Length of a line

If you have two points $A(x_1, y_1)$ and $B(x_2, y_2)$, then the length of the line segment AB is found using the distance formula:

$$\text{Length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

TRY IT

Find the length of the line joining $A(1, 2)$ and $B(4, 6)$.

$$\text{Length} = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Midpoint

To find the middle point between $A(x_1, y_1)$ and $B(x_2, y_2)$, use this formula:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

TRY IT

Find the midpoint of $A(1, 2)$ and $B(4, 6)$.

$$\text{Midpoint} = \left(\tfrac{1+4}{2}, \tfrac{2+6}{2}\right) = (2.5,\ 4)$$

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Equations of linear graphs

Form	What it means	Example
$y = mx + c$	Gradient and y-intercept form	$y = 2x + 3$
$ax + by = c$	General form	$3x + 4y = 12$
$x = k$	A vertical line at $x = k$	$x = 5$
$y = k$	A horizontal line at $y = k$	$y = -2$

Finding the equation of a line

If you know the gradient and a point on the line (x_1, y_1), use this formula:

$$y - y_1 = m(x - x_1)$$

This is called the point-gradient form.

TRY IT

Find the equation of a line with gradient 3 passing through $(-1, 2)$.

$y - 2 = 3\,(x - (-1)) \quad \Rightarrow \quad y - 2 = 3x + 3 \quad \Rightarrow \quad y = 3x + 5$

NOTE

You can also plug the values into $y = mx + c$ to find the equation of the line: $2 = 3(-1) + c$ gives $c = 5$.

Getting information from an equation

You can find the gradient and y-intercept straight from the equation.

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Find the gradient and y-intercept of the line $5x + 4y = 8$.

Rearrange it to the $y = mx + c$ form:

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$4y = -5x + 8 \quad \Rightarrow \quad y = -\frac{5}{4}x + 2$

The gradient is $-\frac{5}{4}$ and the y-intercept is 2.

Parallel and perpendicular lines

Parallel lines have the **same gradient**.

> **TRY IT**
>
> Find the equation of a line parallel to $y = 4x - 1$ that passes through $(1, -3)$.

- The gradient of the given line is 4.
- Use the point $(1, -3)$ and plug into the point-gradient form:

$y + 3 = 4(x - 1) \quad \Rightarrow \quad y + 3 = 4x - 4 \quad \Rightarrow \quad y = 4x - 7$

Perpendicular lines are lines that meet at a right angle ($90°$), like the corners of a square. They have **opposite reciprocal** gradients. If a line has gradient m, then a perpendicular line has gradient:

> $m_{\text{perpendicular}} = -\frac{1}{m}$. **Flip the number, change the sign.**

> **TRY IT**
>
> What is the gradient of a line perpendicular to $2y = 3x + 1$?

First, write it in $y = mx + c$ form: $y = \frac{3}{2}x + \frac{1}{2}$. The gradient is $\frac{3}{2}$, so the perpendicular gradient is $-\frac{2}{3}$ (reciprocal and change sign).

Perpendicular bisector

> **TRY IT**
>
> Find the equation of the perpendicular bisector of the line joining $A(-3, 8)$ and $B(9, -2)$.

A perpendicular bisector cuts the line in half (through the midpoint) at a right angle.

Step 1: find the midpoint.

$\left(\frac{-3+9}{2}, \frac{8+(-2)}{2}\right) = (3, 3)$

Step 2: find the gradient of AB.

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$m = \frac{-2-8}{9-(-3)} = \frac{-10}{12} = -\frac{5}{6}$

Step 3: find the perpendicular gradient. Flip and change the sign: $\frac{6}{5}$.

Step 4: plug the midpoint into the point-gradient form.

$y - 3 = \frac{6}{5}(x - 3) \quad \Rightarrow \quad y = \frac{6}{5}x - \frac{18}{5} + 3 \quad \Rightarrow \quad y = \frac{6}{5}x - \frac{3}{5}$

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