Boolean Logic

The six logic gates and their truth tables, and how to write an expression for a circuit, complete a truth table, draw a circuit from an expression, and build a circuit from a problem statement.

Logic gates

NOTE

Memorising the outputs of all the logic gates is necessary.

NOT

The NOT logic gate is the only logic gate with one input; the rest have two.

Input A	Output X
0	1
1	0

The truth table, which is used to trace the output of logic gates, shows that with input A, it gives output X.

AND

A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

OR

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A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

NAND (NOT AND)

A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

NOR (NOT OR)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

XOR

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

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Logic expressions, logic circuits, problem statements and truth tables

Writing a logic expression for a logic circuit

> **TRY IT**
>
> Write a logic expression for this logic circuit.

- Logic gate 1: NOT A
- Logic gate 2: NOT C

To write down the expressions for the other gates, we will have to combine the expressions of the previous gates.

- Logic gate 3 (logic gate 1 and B): NOT A AND B
- Logic gate 4 (logic gates 3 and 2): (NOT A AND B) OR NOT C

X = (NOT A AND B) OR NOT C

> **NOTE**
>
> Tips:
>
> a. The use of brackets is very important. Make sure they are in the correct places.
>
> b. Work from left to right.

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Completing a truth table from a logic circuit

> **TRY IT**
>
> Complete the truth table from the given logic circuit.

A	B	C	1 (NOT A)	2 (NOT C)	3 (1 AND B)	X
0	0	0	1	1	0	**1**
0	0	1	1	0	0	**0**
0	1	0	1	1	1	**1**
0	1	1	1	0	1	**1**
1	0	0	0	1	0	**1**
1	0	1	0	0	0	**0**
1	1	0	0	1	0	**1**
1	1	1	0	0	0	**0**

This is how you solve a truth table:

1. Count the number of gates and make the required number of columns in the working space (columns 1, 2 and 3).
2. Write the output of each gate one by one. This is where your knowledge of the output of different gates will be useful.
3. The final output will be written in the column labelled X.

Drawing a logic circuit from a logic expression

> **TRY IT**
>
> Draw a logic circuit for this logic expression: Z = (A NAND B) OR NOT (B XOR C)

This is how you make a logic circuit from a logic expression:

1. Solve the brackets first.
2. Remember that the NOT gate only has one input and the others have two.
3. Be careful about the positions of the brackets in the expression.
4. The dots represent joints. They mean that the same value is to be used somewhere else (here B goes to both the NAND and the XOR).

Writing a logic expression from a truth table

> **TRY IT**

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Write down the logic expression representing the table below.

A	B	C	X
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

X = (NOT A AND NOT B AND NOT C) OR (A AND NOT B AND C) OR (A AND B AND C)

Tips to solve this type of question:

- a. The final expression will only consist of the parts where X (the final output) is 1.
- b. Every time X is 1, A, B and C will be connected using AND.
- c. Each output where X is 1 will be connected using OR.
- d. When an input is 1, it will be written as A. When it is 0, it will be written as NOT A.

Building a circuit from a problem statement

TRY IT

A water control system uses a switch and two pressure sensors. The outputs of the switch and sensors are shown in the table.

Sensor or switch	Output of 1	Output of 0
Switch (S1)	On	Off
Pressure sensor (P1)	≥ 3	< 3
Pressure sensor (P2)	≥ 3	< 3

Create a logic circuit that will produce an output (X) of 1 when:

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- the switch S1 is on, **and**
- **either** P1 is less than 3 **or** P2 is less than 3, **but not both**.

All logic gates used must have a maximum of two inputs.

How to solve:

- a. The word in the middle always represents the last logic gate. In this case, "and" represents the AND gate.
- b. Secondly, the question states that we need to have an output of 1. Since we already know that the AND gate only gives an output of 1 when both the inputs are 1, we need to make sure both the inputs are 1.
- c. When the switch S1 is ON, it gives an output of 1, hence we connect it directly to the AND gate.
- d. The question states that either P1 or P2 needs to be less than 3 but not both. By recalling what we have learnt before, we recognise that this is the output achieved by the XOR gate.

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